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High School Geometry Worksheet Bundle: Volume Two

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Geometry: Transformations in the Plane

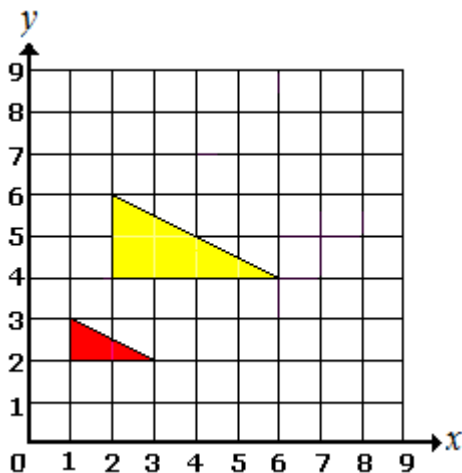
1. What is the rule for a reflection across the y -axis?

- ☐ A. $(x', y') = (-x, y)$
 - ☐ B. $(x', y') = (x, -y)$
 - ☐ C. $(x', y') = (y, x)$
 - ☐ D. $(x', y') = (-x, -y)$
-

2. Which of the following transformations will produce a similar, but not congruent figure?

- ☐ A. rotation
 - ☐ B. reflection
 - ☐ C. dilation
 - ☐ D. translation
-

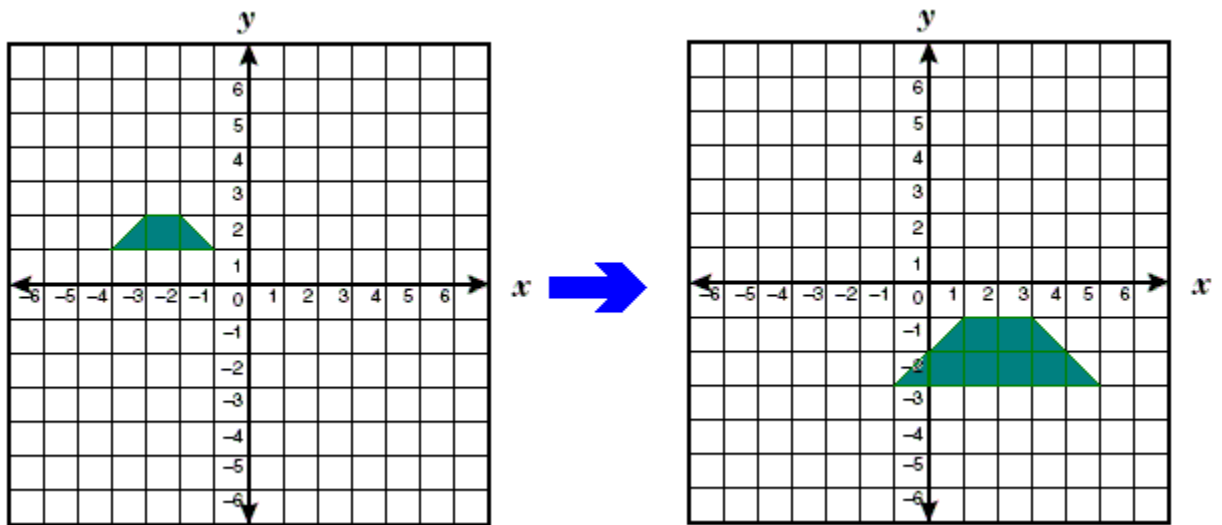
3.



What is the rule for the transformation from the larger polygon to the smaller polygon?

- ☐ A. $(x', y') = \left(\frac{1}{2}x, \frac{1}{2}y\right)$
 - ☐ B. $(x', y') = (3x, 3y)$
 - ☐ C. $(x', y') = \left(\frac{1}{3}x, \frac{1}{3}y\right)$
 - ☐ D. $(x', y') = (2x, 2y)$
-

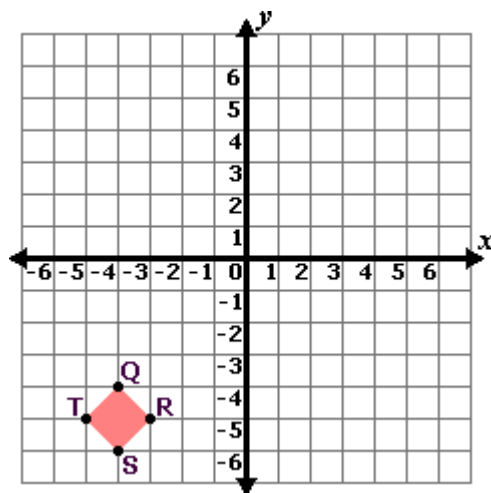
4.



What is the rule for the transformation above?

- ☐ A. $(x', y') = (x + 1, -y - 4)$
- ☐ B. $(x', y') = (-2x + 1, 2y + 4)$
- ☐ C. $(x', y') = (-x - 1, y - 4)$
- ☐ D. $(x', y') = (2x + 7, 2y - 5)$

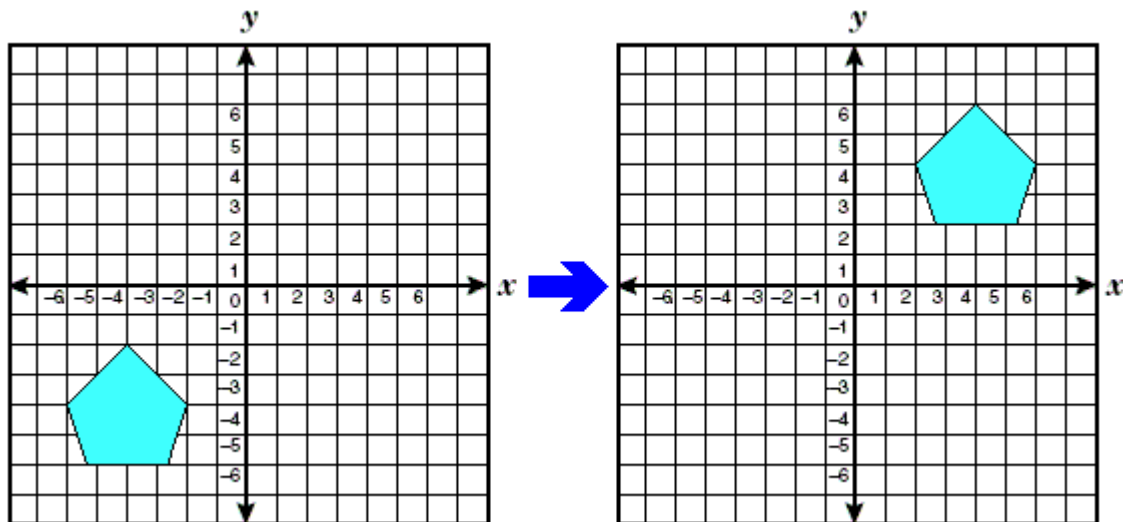
5.



If figure QRST is reflected across the x -axis and then translated 3 units down, which of the following will be the coordinates for point R' ?

- ☐ A. $(0, 5)$
- ☐ B. $(3, -2)$
- ☐ C. $(-3, 2)$
- ☐ D. $(-3, 5)$

6.



What is the rule for the transformation shown above?

- ☐ A. $(x', y') = (x + 4, y + 4)$
- ☐ B. $(x', y') = (x - 4, y - 4)$
- ☐ C. $(x', y') = (x - 8, y - 8)$
- ☐ D. $(x', y') = (x + 8, y + 8)$

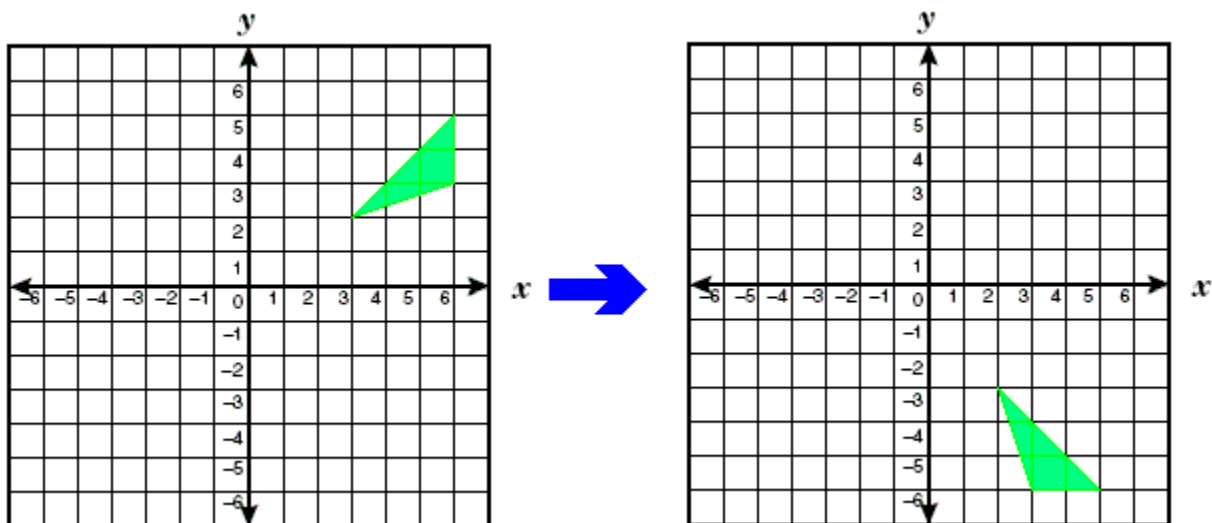
7.

$$\begin{array}{ll} R(-4, -1) & R'(-1, 3) \\ S(-6, 3) & S'(-3, 7) \end{array}$$

Given the points above, which of the following transformations maps RS to R'S'?

- ☐ A. translate horizontally 3 units and vertically -4 units
- ☐ B. translate horizontally -3 units and vertically -4 units
- ☐ C. translate horizontally -3 units and vertically 4 units
- ☐ D. translate horizontally 3 units and vertically 4 units

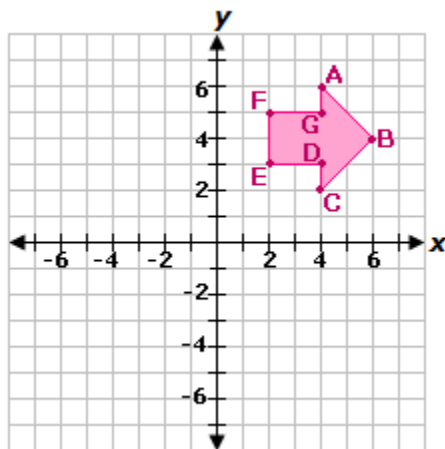
8.



What is the rule for the transformation shown above?

- ☐ A. $(x', y') = (y, x)$
- ☐ B. $(x', y') = (y, -x)$
- ☐ C. $(x', y') = (-y, x)$
- ☐ D. $(x', y') = (-x, -y)$

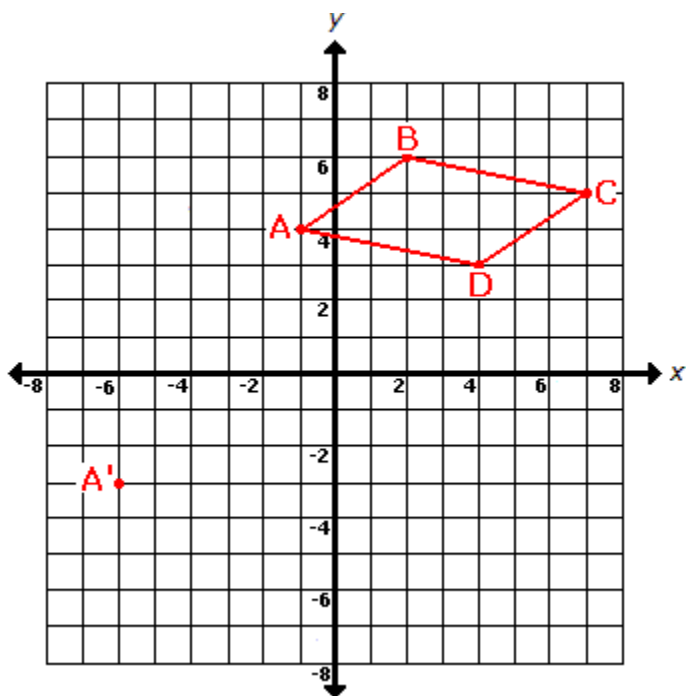
9.



What are the coordinates of C' if A'B'C'D'E'F'G' is a reflection of ABCDEFG across the y-axis?

- ☐ A. $(-4, 6)$
- ☐ B. $(4, -2)$
- ☐ C. $(-2, 4)$
- ☐ D. $(-4, 2)$

10. The parallelogram ABCD is shown below.



If the parallelogram is translated so that A is mapped to A', then what will be the coordinates of C'?

- ☐ A. (1,-1)
- ☐ B. (-2,2)
- ☐ C. (2,-2)
- ☐ D. (2,-1)

Answers: Transformations in the Plane

1. A
2. C
3. A
4. D
5. C
6. D
7. D
8. B
9. D
10. C

Explanations

1. In a reflection across the y -axis, for each point of the polygon, the x -value changes sign while the y -value stays the same.

Thus $x' = -x$ and $y' = y$.

2. Two figures are congruent if they have corresponding angles of equal measure and sides of equal measure. A dilation is a transformation which transforms each line to a parallel line whose length is a fixed multiple of the length of the original line. Since a dilation does not keep the lengths of the sides the same, figures obtained by **dilation** are similar, and not congruent.

3. Notice that both figures have the same shape and orientation, but they have different sizes. A dilation is the only type of transformation that produces a change in size.

Choose a point on the larger polygon, such as (2, 4). Then, find the corresponding point, after the object has been translated, on the smaller polygon.

The point (2, 4) on the larger polygon corresponds to the point (1, 2) on the smaller polygon.

Thus, the values of the x - and y -coordinates were both halved during the transformation. Therefore, the following is true.

$$x' = \frac{1}{2}x \text{ and } y' = \frac{1}{2}y$$

So, the rule for the transformation is shown below.

$$(x', y') = \left(\frac{1}{2}x, \frac{1}{2}y\right)$$

4. The transformation shows a dilation and a translation. The transformation can be written in two different ways, one in which the dilation is done first and one in which the translation is done first. In this

explanation of the transformation, the dilation is done first.

Start by determining the scale factor of the dilation. The length of the bottom of the original polygon is 3 units. The length of the bottom of the translated polygon is 6 units. So, the scale factor is two and $(x', y') = (2x, 2y)$.

Now, choose a point on the original polygon, such as $(-1, 1)$. Then, find the corresponding point after a dilation with a scale factor of two.

So, the point $(-1, 1)$ should be at point $(-2, 2)$ after the dilation. Now, determine the translation needed to get from the dilated point $(-2, 2)$ to its final resting place at the point $(5, -3)$. The translation can be found by subtracting the dilated coordinates from the final coordinates.

For x : $5 - (-2) = 7$.

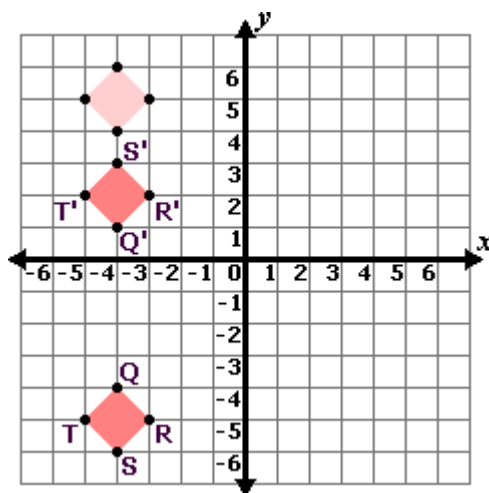
For y : $-3 - 2 = -5$.

So, the rule for the translation after the dilation is $(x', y') = (x + 7, y - 5)$.

The combined transformation for both operations (dilation and translation) is $(x', y') = (2x + 7, 2y - 5)$.

5. When point $P(x, y)$ is reflected across the x -axis and translated 3 units down, the translation point, P' , is located at $(x, y - 3)$, where the y -coordinate changes signs and 3 units are subtracted from the y -coordinate to move down. The x -coordinate remains the same.

Thus, the coordinates of point R reflected across the x -axis and translated 3 units down are $R'(-3, 2)$.



6. Choose a corner of the object, such as $(-4, -2)$. Then see what point that corner is on after the object has been translated.

The corner on point $(-4, -2)$ is on point $(4, 6)$ after the translation.

The object moved 8 units horizontally on the x -axis (the 8 is positive since it moved to the right on the x -axis). Therefore, $x' = x + 8$.

It also moved 8 units vertically on the y -axis (the 8 is positive since it moved up the y -axis). Therefore, $y' =$

$$y + 8.$$

So, the rule for the transformation is $(x', y') = (x + 8, y + 8)$.

7. The point $R(-4, -1)$ maps to $R'(-1, 3)$.

So, R is shifted 3 units horizontally on the x -axis and is shifted 4 units vertically on the y -axis.

Therefore, $(x', y') = (x + 3, y + 4)$ which indicates the map **translated horizontally 3 units and vertically 4 units**.

8. For any given angle, α , the new coordinates for a counterclockwise rotation about the origin of α is the following.

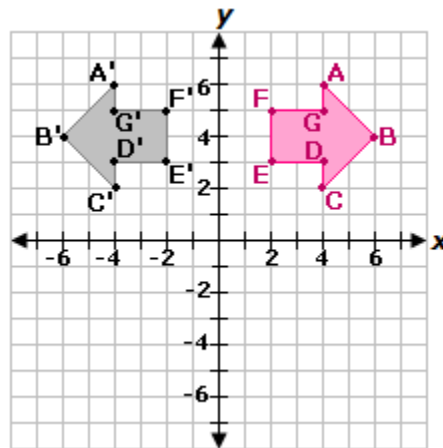
$$(x', y') = (x \cos \alpha - y \sin \alpha, x \sin \alpha + y \cos \alpha)$$

The polygon has been transformed by a counterclockwise rotation of 270° about the origin.

$$\begin{aligned} (x', y') &= (x \cos 270^\circ - y \sin 270^\circ, x \sin 270^\circ + y \cos 270^\circ) \\ &= (x(0) - y(-1), x(-1) + y(0)) \\ &= (y, -x) \end{aligned}$$

9. When point $P(x, y)$ is reflected over the y -axis, the point P' is located at $(-x, y)$, where only the sign of the x -coordinate changes.

Thus, the coordinates of the reflection across the y -axis of point C are $C'(-4, 2)$.



10. The point A at $(-1, 4)$ is mapped to point A' at $(-6, -3)$.

The change in the x -coordinate is $-6 - (-1) = -5$ or 5 units to the left.

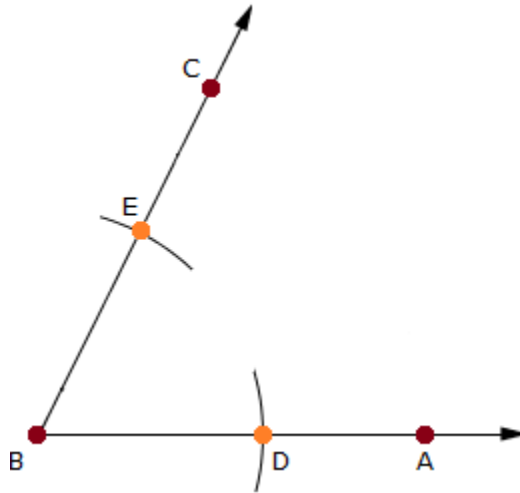
The change in the y -coordinate is $-3 - 4 = -7$ or 7 units down.

Next, find C' . The coordinates of C are at $(7, 5)$. Shifting the x -coordinate 5 units to the left gives, $x' = 7 - 5 = 2$. Shifting the y -coordinate 7 units down gives, $y' = 5 - 7 = -2$.

So, the coordinates of C' are $(2, -2)$.

Geometry: Geometric Constructions

1. Below is a stage in the construction of a bisector of $\angle ABC$.



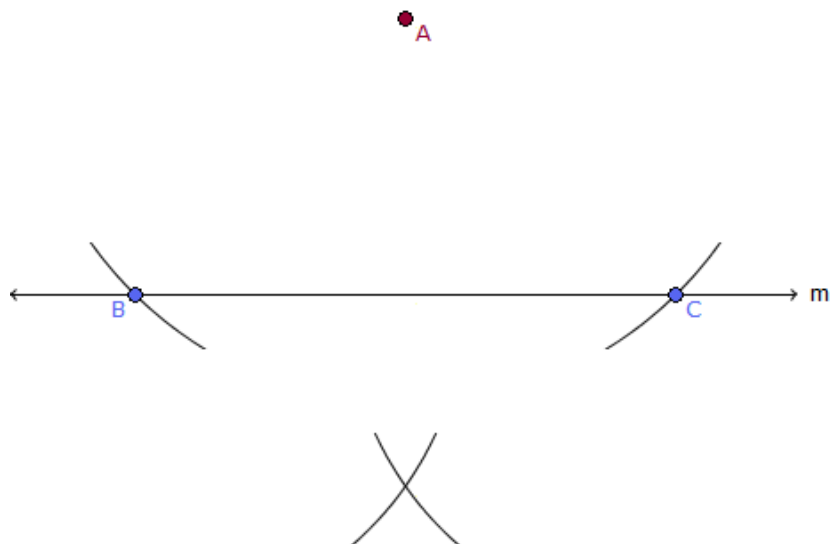
Which of the following describes the next step in the construction?

- ☐ A. Draw an arc intersecting \overrightarrow{BA} centered at point D using any compass width.
- ☐ B. Draw an arc intersecting \overrightarrow{BA} centered at point E using any compass width.
- ☐ C. Draw intersecting arcs within the interior of $\angle ABC$ from points D and E without changing the compass width.
- ☐ D. Set the compass width to the distance between points D and E.

2. What is the first step to construct a line parallel to \overleftrightarrow{AB} passing through an external point C, using only a compass and a straightedge?

- ☐ A. Draw a line through point C that intersects \overleftrightarrow{AB} and mark the point of intersection.
- ☐ B. Place the compass needle on point C and using sufficient compass width mark two arcs across \overleftrightarrow{AB} .
- ☐ C. Place the compass needle at any point on \overleftrightarrow{AB} and set the compass width to point C.
- ☐ D. Make two arcs through point C centered at points A and B.

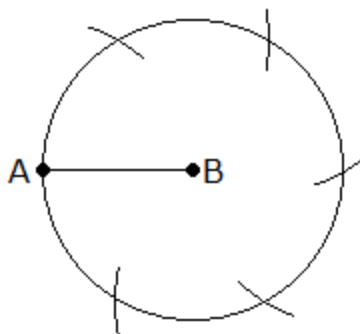
3. When Jackson walked into Geometry class, the following diagram was on the chalk board.



Which of the following constructions has a step that would involve this diagram?

- ☐ A. construction of a line parallel to a given line
 - ☐ B. construction of a perpendicular bisector
 - ☐ C. construction of a perpendicular to a line through a point not on the line
 - ☐ D. construction of a perpendicular to a line through a point on the line
-

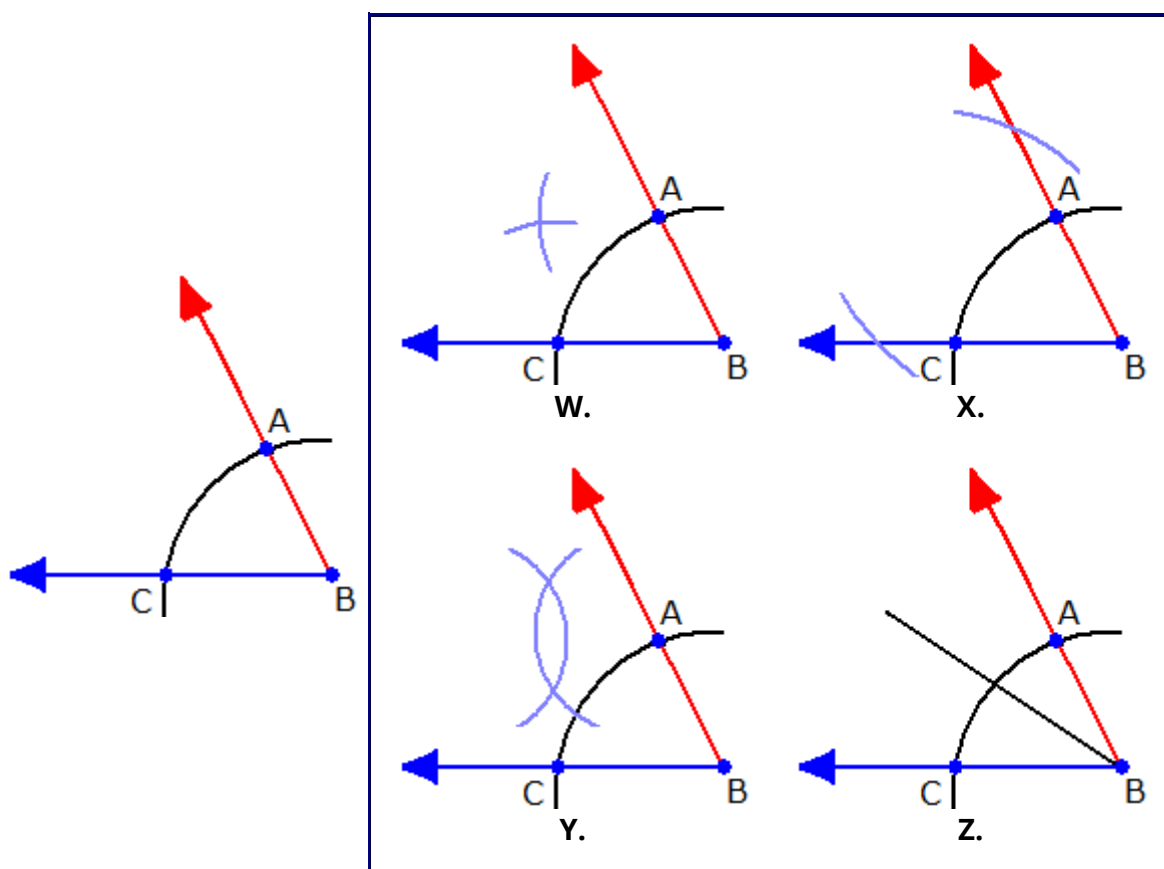
4. Sally used a ruler to draw the segment AB. She then opened her compass to the length of AB and drew a circle. Keeping the compass open to the same length, she marked off equal parts along the circle as shown in the following figure.



Using some or all of the points of intersection and point A, which of the following regular polygons could she construct?

- ☐ A. hexagon
- ☐ B. decagon
- ☐ C. pentagon
- ☐ D. square

5. Given $\angle ABC$ below, which would be the next step when constructing the bisector of $\angle ABC$?

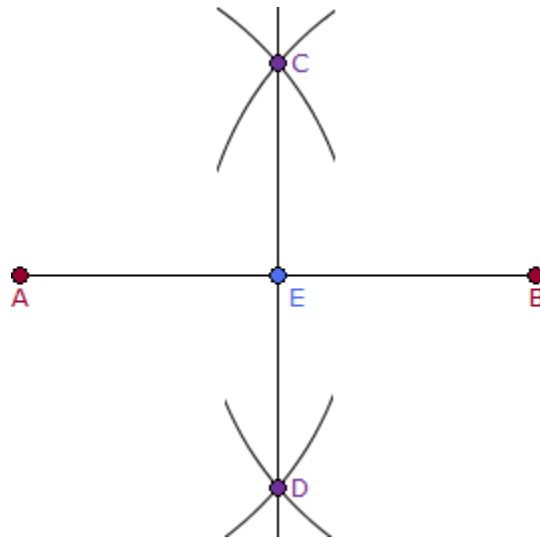


- ☐ A. W
- ☐ B. Y
- ☐ C. X
- ☐ D. Z

6. To copy an angle using only a compass and a straightedge, begin by marking the vertex of the new angle. Then, draw a ray from the vertex, which will be one of the legs of the new angle. What is the next step in the construction?

- ☐ A. Place the compass needle at any point on the leg of the new angle, set the compass width to the vertex of the angle, and make an arc in the angle's interior.
- ☐ B. Place the compass needle on one of the legs of the original angle, and draw an arc on the leg of the new angle using a convenient compass width.
- ☐ C. Set the compass to a convenient width and draw an arc from the vertex of the original angle intersecting both legs of the original angle.
- ☐ D. Place the compass needle on one of the legs of the original angle and draw an arc on the other leg of the angle using sufficient compass width.

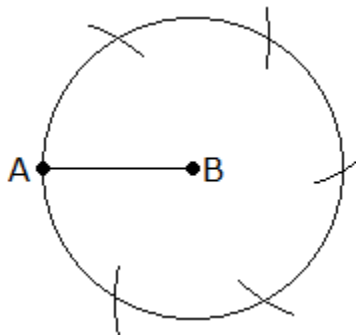
7. Line CD is constructed by making arcs centered at point A and point B with the same compass width.



Which of the following equations is not necessarily true?

- ☐ A. $AB = CD$
 - ☐ B. $AE = EB$
 - ☐ C. $AD = DB$
 - ☐ D. $AC = CB$
-

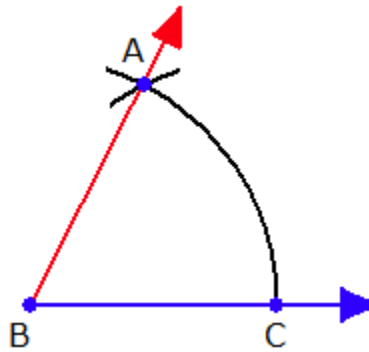
8. Sally used a ruler to draw the segment AB. She then opened her compass to the length of AB and drew a circle. Keeping the compass open to the same length, she marked off equal parts along the circle as shown in the following figure.



Using some or all of the points of intersection and point A, which of the following regular polygons could she construct?

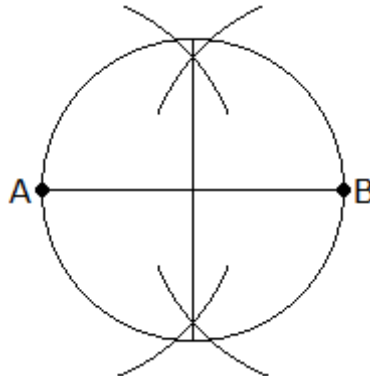
- ☐ A. square
- ☐ B. pentagon
- ☐ C. decagon
- ☐ D. equilateral triangle

9. The figure below illustrates constructing _____.



- ☐ A. a bisector of an angle
 - ☐ B. a congruent angle
 - ☐ C. a right angle
 - ☐ D. an obtuse angle
-

10. Brian used a ruler to draw the segment AB. He then constructed a bisector of AB and used it to create another diameter as shown in the following figure.



Using the points of intersection of the circle and the diameters, which of the following regular polygons could be formed?

- ☐ A. hexagon
- ☐ B. equilateral triangle
- ☐ C. pentagon
- ☐ D. square

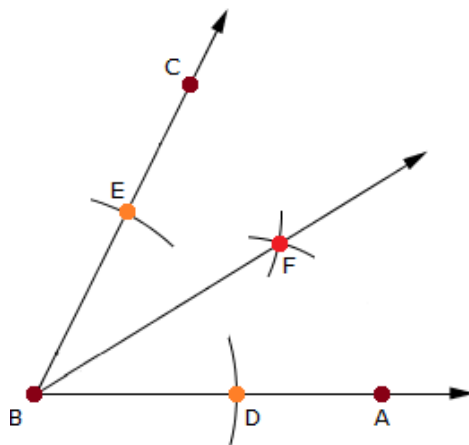
Answers: Geometric Constructions

1. C
2. A
3. C
4. A
5. A
6. C
7. A
8. D
9. B
10. D

Explanations

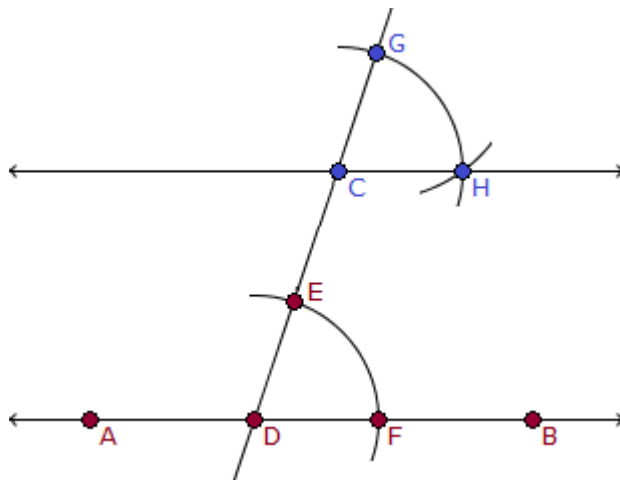
1. The steps for constructing the bisector of an angle using only a compass and a straightedge are shown below.

1. Place the compass needle on the vertex of $\angle ABC$.
2. Set the compass to any convenient width.
3. Draw an arc intersecting both rays of the angle. Mark and label the points of intersection, D and E.
4. Move the compass needle to point D. If required, adjust the compass width at this point. Draw an arc within the interior of $\angle ABC$.
5. Without changing the width of the compass, move its needle to point E and draw another arc that crosses the arc drawn from point D.
6. Mark the point of intersection of the arcs centered at point D and point E. Label the intersection point F.
7. Draw a straight line from vertex B passing through point F.
- 8.



2. The steps for constructing a line parallel to \overleftrightarrow{AB} and passing through point C not on \overleftrightarrow{AB} using only a compass and a straightedge are show below.

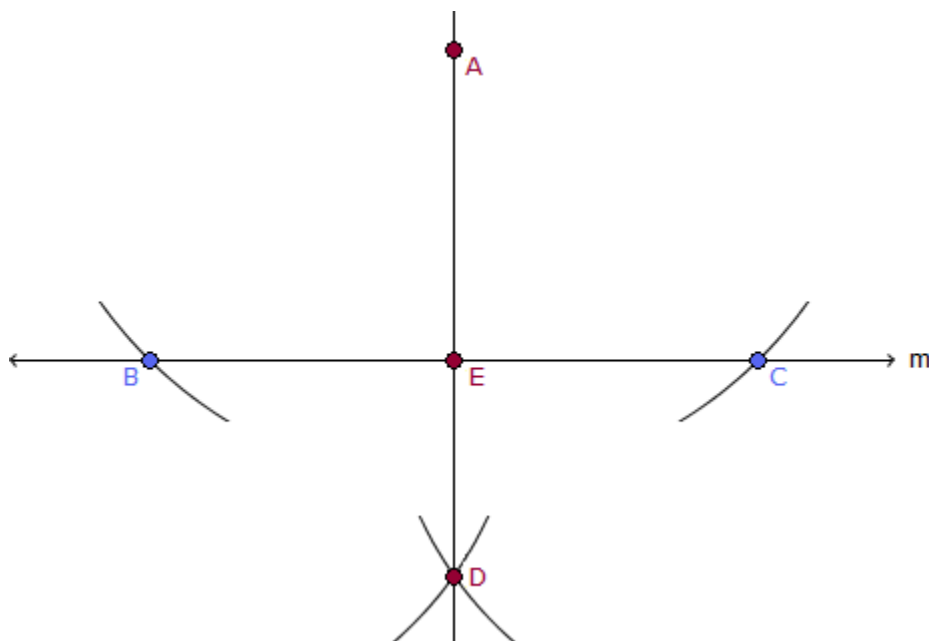
1. Draw a line through point C that intersects \overleftrightarrow{AB} . Label the intersection of the lines point D.
2. Place the compass needle on point D, and set the compass width to approximately half the distance between points D and C. Draw an arc intersecting both lines and label the points of intersection E and F.
3. Keeping the compass width the same, place the compass needle on point C and draw an arc similar to the one drawn from point D. Label the point of intersection G.
4. Place the compass needle on point E and set the compass width to the length of EF.
5. Move the compass needle to point G and draw an arc intersecting the upper arc. Label the point of intersection H.
6. Draw a line through points C and H. Line CH is parallel to \overleftrightarrow{AB} .
- 7.



3. The steps for constructing the a line perpendicular to a given line through a point not on the given line using only a compass and a straightedge are show below.

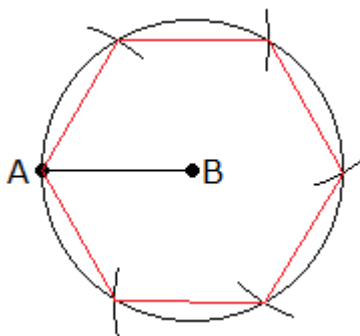
1. Start with line m and a point A not on the line m.
2. Place the compass needle on the external point A.
3. Set the width of the compass to be greater than the distance from point A to line m.
4. Draw an arc across line m on each side of point A. Label these points B and C.
5. Set the compass needle on point B and set the width of the compass to be greater than half the length of BC.
6. From point B, make an arc on the opposite side of line m from point A.
7. Without changing the compass width, make an arc from point C that intersects the arc drawn from point B. Label the point of intersection D.
8. Draw a line through points A and D. \overleftrightarrow{AD} is perpendicular to line m at point E.

9.



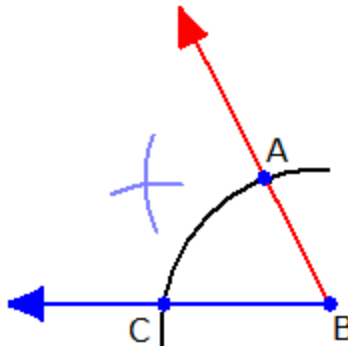
So, the diagram that was on the chalk board is from the **construction of a perpendicular to a line through a point not on the line**.

4. Connecting all points of intersection will form a regular **hexagon** as shown in the following figure.



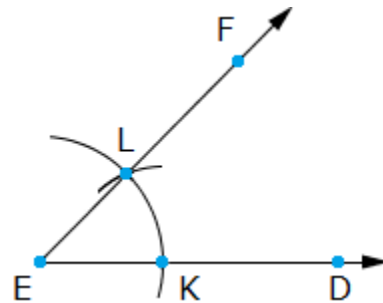
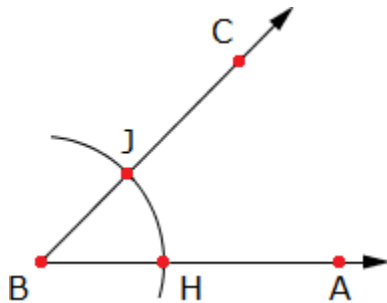
5. The next step in the bisection of $\angle ABC$ would be to place the compass on point C and draw an arc in the middle of the angle. Then, move the compass to point A and draw the same arc so that it intersected the first one.

This is illustrated by choice **W**.



6. The steps for constructing the copy of an angle using only a compass and a straightedge are show below.

1. Mark the point that will be the vertex of the new angle and label it point E.
2. Draw a ray from point E in any direction, with any length. This ray will be one of the sides of the new angle.
3. Place the compass needle on the vertex of the original angle and adjust the width of the compass to a convenient size.
4. Use the compass to draw an arc intersecting both rays of the original angle at two points, H and J.
5. Move the compass (without changing the width) to point E and make an arc intersecting the existing ray of the angle. Mark the intersection point and label it point K.
6. Set the compass needle on the original angle at point H and set its width to the length of HJ.
7. Move the compass needle to point K on the new angle and draw an arc intersecting the previous arc. Mark the intersection point and label it point L.
8. Draw a ray from point E through point L.
- 9.

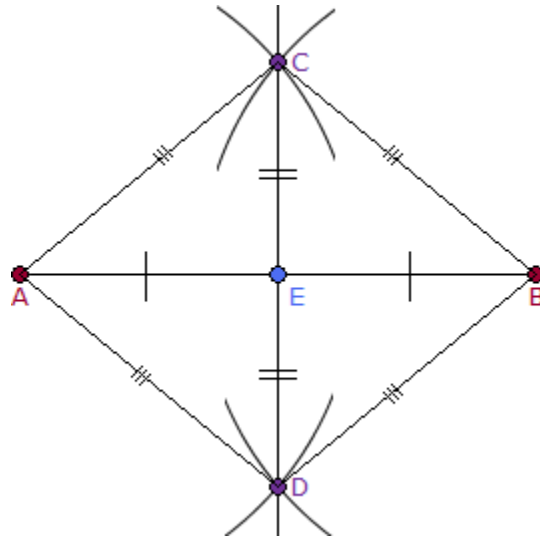


7. The construction described in the problem follows that of the perpendicular bisector of AB using only a compass and a straightedge.

Since \overleftrightarrow{CD} is a perpendicular bisector of AB, $AE = EB$.

Since any point on a perpendicular bisector is equidistant from each endpoint of the line segment, $AC = CB$ and $AD = DB$.

Therefore, the length of AB does not necessarily equal the length of CD.

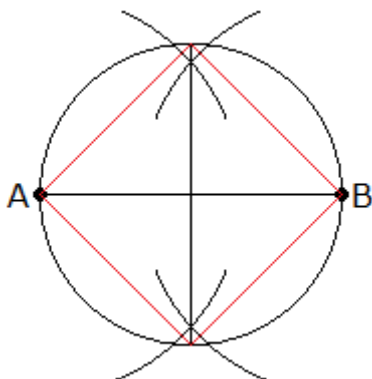


The sequence of diagrams illustrates the construction of an angle bisector:

- Diagram 1:** An angle is formed by two rays originating from point E . One ray is horizontal and points to the right, passing through point F . The other ray points upwards and to the right, passing through point D . A red arrow is on the ray ED .
- Diagram 2:** A black arc is drawn with center E , intersecting the horizontal ray at point F and the upper ray at point D .
- Diagram 3:** A second black arc is drawn with center F , intersecting the horizontal ray at point F and the upper ray at point D . The two arcs intersect at a point above the angle.
- Diagram 4:** The two arcs intersect at a point. A red arrow is drawn from point E through this intersection point, bisecting the angle.
- Diagram 5:** The construction is repeated for a second angle. A horizontal ray originates from point B and passes through point C . A black arc is drawn with center B , intersecting the ray at point C .
- Diagram 6:** A second black arc is drawn with center C , intersecting the ray at point C and the upper ray at point A . The two arcs intersect at a point above the angle. A red arrow is drawn from point B through this intersection point, bisecting the angle.

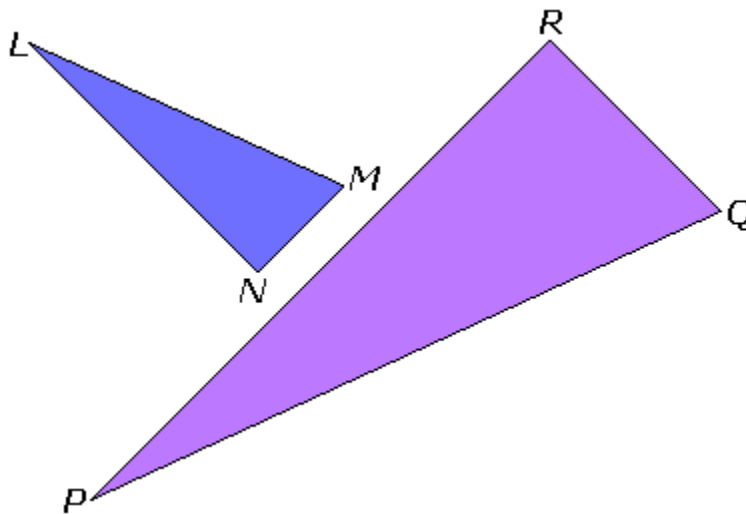
The last step of the construction matches the given illustration. Therefore, this is the construction of a **congruent angle**.

10. Connecting the points of intersection of the diameters and the circle will form a **square** as shown in the following figure.



Geometry: Right Triangle Trigonometry

1.



**Picture not drawn to scale.*

Triangle LMN is similar to triangle PQR . The length of PQ is twice the length of LM .

If the sine of angle L is $\frac{8}{17}$, what is the sine of angle P ?

- ☐ A. $\frac{17}{8}$
- ☐ B. $\frac{16}{17}$
- ☐ C. $\frac{25}{17}$
- ☐ D. $\frac{8}{17}$

2. Angles A and B are complementary angles in a right triangle. The value of $\cos(A)$ is $\frac{4}{5}$. What is the value of $\sin(A)$?

- ☐ A. $\frac{5}{3}$
 - ☐ B. $\frac{3}{5}$
 - ☐ C. $\frac{4}{3}$
 - ☐ D. $\frac{5}{4}$
-

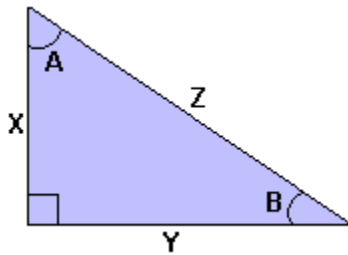
3. Which of the following functions is equal to $\cos(90^\circ - \alpha)$?

- ☐ A. $\tan(90^\circ - \alpha)$
 - ☐ B. $\sec(\alpha)$
 - ☐ C. $\csc(90^\circ - \alpha)$
 - ☐ D. $\sin(\alpha)$
-

4. Which of the following functions is equal to $\sin(76^\circ)$?

- ☐ A. $\cos(14^\circ)$
- ☐ B. $\csc(76^\circ)$
- ☐ C. $\csc(14^\circ)$
- ☐ D. $\cos(76^\circ)$

5.

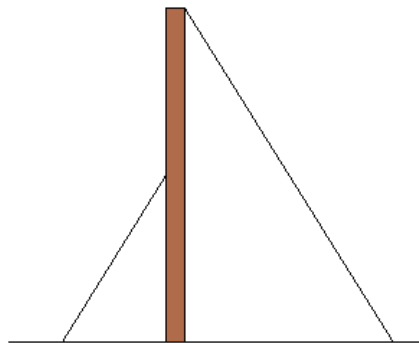


Note: Picture is not drawn to scale.

If $X = 9$ feet, $Y = 12$ feet, and $Z = 15$ feet, what is the cosine of $\angle B$?

- ☐ A. $\frac{3}{5}$
- ☐ B. $\frac{3}{4}$
- ☐ C. $\frac{4}{3}$
- ☐ D. $\frac{4}{5}$

6.

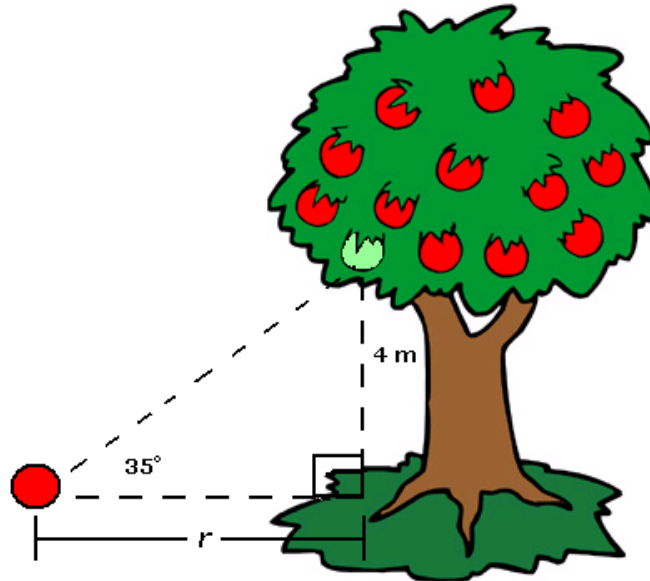


Two wires are attached to a pole and create similar triangles with the ground. The longer wire is attached to the ground 32 feet from the base of the pole and the shorter wire is attached to the ground 16 feet from the base of the pole.

If the cosine of the angle formed by the shorter wire and the ground is $\frac{8}{41}$, what is the length of the longer wire?

- ☐ A. 164 feet
- ☐ B. 246 feet
- ☐ C. 123 feet
- ☐ D. 82 feet

7. An apple that was 4 meters off the ground was blown off a tree. The angle between the final position of the apple and the original position of the apple was 35° .

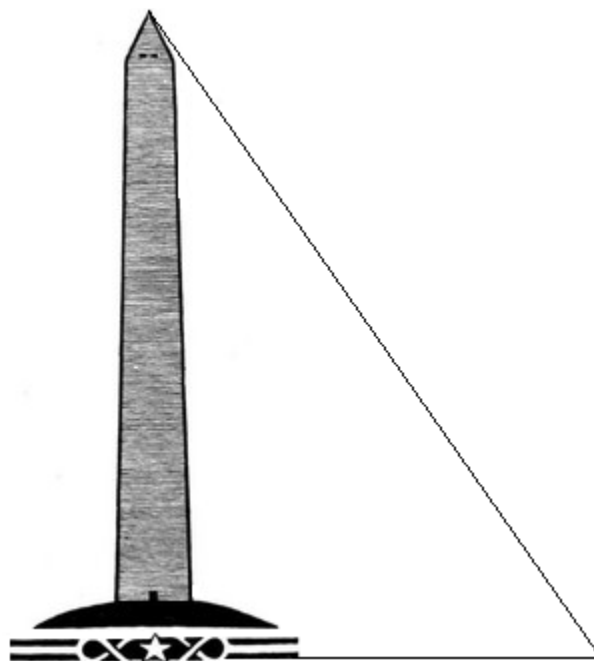


Note: picture not drawn to scale

Which equation can be used to find the horizontal distance, r , that the apple was displaced?

- ☐ A. $\sin 35^\circ = \frac{4 \text{ m}}{r}$
- ☐ B. $\sin 35^\circ = \frac{r}{4 \text{ m}}$
- ☐ C. $\cos 35^\circ = \frac{4 \text{ m}}{r}$
- ☐ D. $\tan 35^\circ = \frac{4 \text{ m}}{r}$

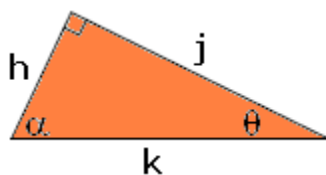
8.



Amy is standing 65 meters from the base of the Washington Monument in Washington, DC. She estimates that the angle of elevation to the top of the building is 60° . One of her friends is at the top of the building. What is the distance between Amy and her friend? (Assume the monument meets the ground at a right angle.)

- ☐ A. 113 meters
 - ☐ B. 75 meters
 - ☐ C. 33 meters
 - ☐ D. 130 meters
-

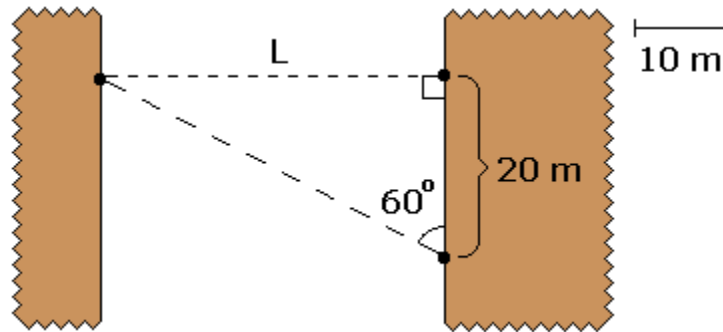
9.



If $\alpha = 65^\circ$ and $h = 5$ mm, what is the value of k to the nearest tenth of a millimeter?

- ☐ A. 11.8 mm
- ☐ B. 10.7 mm
- ☐ C. 5.5 mm
- ☐ D. 11.3 mm

10.



An engineer wants to build a bridge across a gorge as shown in the figure above. To calculate the length the bridge will need to be, L , she chooses two points directly across the gorge from each other to form one side of a right triangle and then another point that is 20 m apart from one of the first points to make the other side of the right triangle. She calculates the angle opposite the longer side to be 60° .

What is the distance across the gorge, L ? Round to the nearest hundredth of a meter.

- ☐ A. 34.64 m
- ☐ B. 40 m
- ☐ C. 10 m
- ☐ D. 17.32 m

Answers: Right Triangle Trigonometry

1. D
2. B
3. D
4. A
5. D
6. A
7. D
8. D
9. A
10. A

Explanations

1. The trigonometry ratios of similar triangles are equivalent for corresponding angles.

Since angle L corresponds to angle P , the sine of angle L is equal to the sine of angle P .

So, the sine of angle P is $\frac{8}{17}$.

2. Given a right triangle with complementary angles A and B , the following is true.

$$\cos(A) = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

In this problem, $\cos(A) = \frac{4}{5}$. Use the following ratio to find $\sin(A)$.

$$\sin(A) = \frac{\text{opposite side}}{\text{hypotenuse}}$$

Apply the Pythagorean theorem to find the value of the hypotenuse, then use the trigonometric ratios to solve for $\sin(A)$.

$$a^2 + b^2 = c^2$$

$$a^2 + 4^2 = 5^2$$

$$a^2 = 9$$

$$b = 3$$

The value of the hypotenuse was given in the problem, and the value of the opposite side was solved for; therefore, $\sin(A) = \frac{3}{5}$.

3. The co-functions of complementary angles are equal. If α is an acute angle, then the following relationship is true.

$$\cos(90^\circ - \alpha) = \sin(\alpha)$$

4. The co-functions of complementary angles are equal. If α is an acute angle, then the following relationship is true.

$$\sin(90^\circ - \alpha) = \cos(\alpha)$$

Subtract 76° from 90° to find the complementary angle of 76° .

$$90^\circ - 76^\circ = 14^\circ$$

Therefore, $\sin(76^\circ) = \cos(14^\circ)$.

5. In the right triangle, the cosine of $\angle B$ is the ratio of the side adjacent $\angle B$ to the hypotenuse.

$$\begin{aligned}\cos(B) &= \frac{\text{adjacent}}{\text{hypotenuse}} \\ &= \frac{Y}{Z} \\ &= \frac{12 \text{ feet}}{15 \text{ feet}} \\ &= \frac{4}{5}\end{aligned}$$

6. The trigonometric ratios of similar triangles are equivalent.

The cosine of the angle formed by the shorter wire and the ground is $\frac{8}{41}$.

So, the cosine of the angle formed by the longer wire and the ground is also $\frac{8}{41}$.

Use this fact to find the length of the longer wire, x .

$$\begin{aligned}\frac{8}{41} &= \frac{32 \text{ feet}}{x} \\ 8x &= (41)(32 \text{ feet}) \\ 8x &= 1,312 \text{ feet} \\ x &= 164 \text{ feet}\end{aligned}$$

So, the length of the longer wire is **164 feet**.

7. Since the angle between the final and the original positions of the apple, and the distance the apple fell are known, the *tangent* equation should be used.

$$\begin{aligned}\tan \alpha &= \frac{\text{opposite}}{\text{adjacent}} \\ \tan 35^\circ &= \frac{4 \text{ m}}{r}\end{aligned}$$

8. Let α be the angle of elevation in the right triangle. Therefore, the following is true.

$$\cos \alpha = \frac{\text{adjacent}}{\text{hypotenuse}}$$

The question gives that the hypotenuse represents the distance between the two friends. Label this variable d .

It also gives that the side adjacent to α is equal to 65 m and $\alpha = 60^\circ$.

Therefore, the following is true.

$$\cos(60^\circ) = \frac{65 \text{ m}}{d}$$

Now, solve for d .

$$\begin{aligned} d &= \frac{65 \text{ m}}{\cos(60^\circ)} \\ &= 130 \text{ m} \end{aligned}$$

The friends are **130 meters** apart.

9. Given a right triangle and an acute angle α of the right triangle, the following is true.

$$\sin(\alpha) = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{j}{k}$$

$$\cos(\alpha) = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{h}{k}$$

$$\tan(\alpha) = \frac{\text{opposite}}{\text{adjacent}} = \frac{j}{h}$$

Thus, k can be found by the following steps.

$$\cos(65^\circ) = \frac{h}{k}$$

$$k = \frac{h}{\cos(65^\circ)}$$

$$k = \frac{5 \text{ mm}}{\cos(65^\circ)}$$

$$k = 11.8 \text{ mm}$$

10. Use the tangent function to solve for the distance.

$$\tan(60^\circ) = \frac{L}{20 \text{ m}}$$

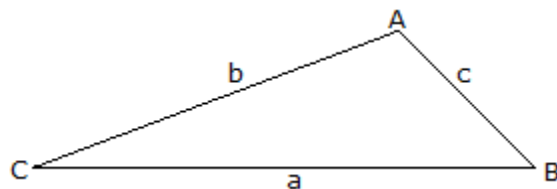
$$(20 \text{ m})(\tan(60^\circ)) = L$$

$$34.64 \text{ m} \approx L$$

Therefore, the bridge will need to be approximately **34.64 m** in length.

Geometry: Law of Sines and Law of Cosines

1.

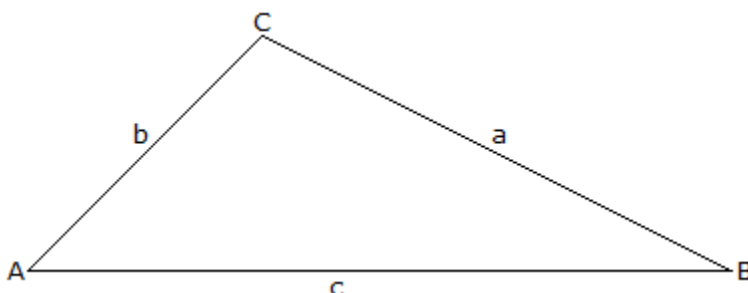


Note: Figure not drawn to scale.

In the triangle shown above, $m\angle B = 49^\circ$, $a = 20$ cm, and $c = 10$ cm. What is the approximate length of side b ?

- ☐ A. 22.33 cm
 - ☐ B. 237.58 cm
 - ☐ C. 19.2 cm
 - ☐ D. 15.41 cm
-

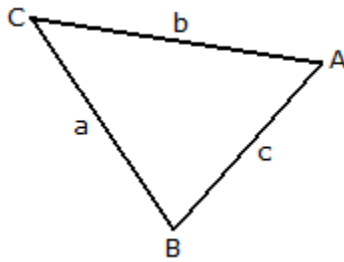
2. The law of cosines can be proved using the Pythagorean theorem.



Given triangle ABC, which statement below correctly uses the Pythagorean theorem in the proof of the law of cosines?

- ☐ A. $a^2 = (b \cos(A))^2 + (c - b \cos(A))^2$
- ☐ B. $a^2 = (b \sin(A))^2 + (b \cos(A))^2$
- ☐ C. $a^2 = (b \sin(A))^2 + (c + b \cos(A))^2$
- ☐ D. $a^2 = (b \sin(A))^2 + (c - b \cos(A))^2$

3.

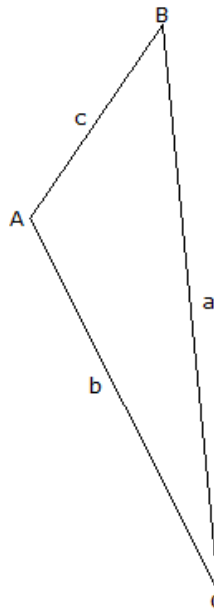


Note: Figure not drawn to scale.

In the triangle shown above, $m\angle A = 42^\circ$, $b = 11$ m, and $c = 19$ m. What is the approximate length of side a ?

- ☐ A. 13.09 m
 - ☐ B. 21.92 m
 - ☐ C. 171.37 m
 - ☐ D. 18.07 m
-

4.

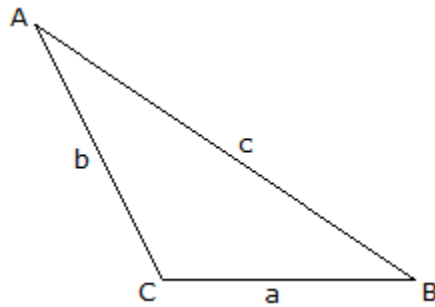


Note: Figure not drawn to scale.

In the triangle shown above, $m\angle A = 114^\circ$, $m\angle C = 22^\circ$, and $a = 50$ ft. What is the approximate length of side b ?

- ☐ A. 34.34 ft
- ☐ B. 38.02 ft
- ☐ C. 61.62 ft
- ☐ D. 1.34 ft

5.



Note: Figure not drawn to scale.

In the triangle shown above, $a = 7$ m, $b = 14$ m, and $c = 18$ m. What is the approximate measure of angle C?

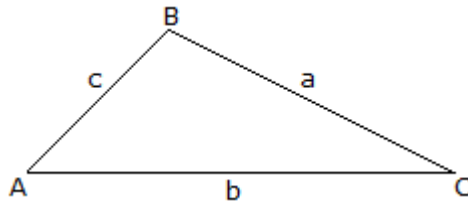
- ☐ A. 63.71°
 - ☐ B. 113.77°
 - ☐ C. 116.29°
 - ☐ D. 134.62°
-

6. Two forces acting on an object form a right angle. Force A is 25 pounds, force B is 60 pounds, and the resultant force is 65 pounds.

What is the measure of the angle formed by the resultant force and force A? (Round to the nearest degree.)

- ☐ A. 25°
- ☐ B. 69°
- ☐ C. 23°
- ☐ D. 67°

7.

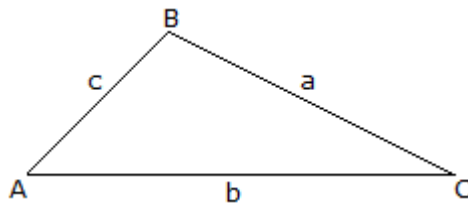


Note: Figure not drawn to scale.

In the triangle shown above $m\angle A = 49^\circ$, $a = 14$ cm, and $c = 8$ cm. What is the approximate measure of angle C?

- ☐ A. 35.94°
 - ☐ B. 25.55°
 - ☐ C. 154.45°
 - ☐ D. 40.16°
-

8.



Note: Figure not drawn to scale.

In the triangle shown above $m\angle B = 106^\circ$, $m\angle A = 44^\circ$, and $b = 12$ in. What is the approximate length of side c?

- ☐ A. 8.14 in
 - ☐ B. 23.07 in
 - ☐ C. 3.89 in
 - ☐ D. 6.24 in
-

9. Two forces are acting on an object. Force X is a 36-pound force pulling upward, and force Y is a 48-pound force pulling to the right.

If the resultant force is 60 pounds, what is the approximate measure of the angle formed by force X and the resultant force? (Round to the nearest degree.)

- ☐ A. 37°
 - ☐ B. 53°
 - ☐ C. 48°
 - ☐ D. 90°
-

10. The law of sines states that if ABC is a triangle with sides a, b, and c, then the following is true.

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

In order to prove the law of sines, what must first be constructed in triangle ABC?

- ☐ **A.** an altitude of triangle ABC
- ☐ **B.** a median of triangle ABC
- ☐ **C.** a perpendicular bisector of triangle ABC
- ☐ **D.** an angle bisector of triangle ABC

Answers: Law of Sines & Cosines

1. D
2. D
3. A
4. B
5. B
6. D
7. B
8. D
9. B
10. A

Explanations

1. The law of cosines states that $b^2 = a^2 + c^2 - 2ac \cos(B)$.

The question gives that $m\angle B = 49^\circ$, $a = 20$ cm, and $c = 10$ cm.

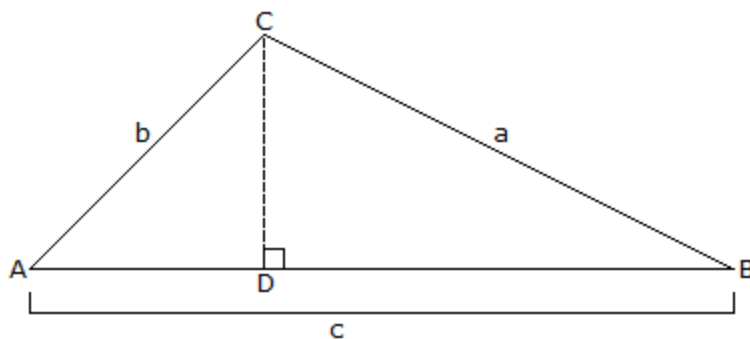
Evaluate the formula with the given information to find the approximate length of side b.

$$\begin{aligned}b^2 &= a^2 + c^2 - 2ac \cos(B) \\b^2 &= (20 \text{ cm})^2 + (10 \text{ cm})^2 - 2(20 \text{ cm})(10 \text{ cm})\cos(49^\circ) \\\sqrt{b^2} &\approx \sqrt{237.58 \text{ cm}^2} \\b &\approx 15.41 \text{ cm}\end{aligned}$$

2. The law of cosines states that if ABC is a triangle with sides a, b, and c, then the following statements are true.

$$\begin{aligned}a^2 &= b^2 + c^2 - 2bc \cos(A) \\b^2 &= a^2 + c^2 - 2ac \cos(B) \\c^2 &= a^2 + b^2 - 2ab \cos(C)\end{aligned}$$

Given triangle ABC, construct an altitude of the triangle. In this case, altitude CD was constructed, as shown below.



Two right triangles, ACD and BCD, were formed. From these two right triangles, the following statements can be made.

$$\cos(A) = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{DA}{b}$$

$$\sin(A) = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{CD}{b}$$

Thus, $DA = b \cos(A)$, $CD = b \sin(A)$, and $DB = c - b \cos(A)$.

Triangle BCD is a right triangle with side a as the hypotenuse. Apply the Pythagorean theorem.

$$\begin{aligned} a^2 &= (CD)^2 + (DB)^2 \\ &= (b \sin(A))^2 + (c - b \cos(A))^2 \\ &= b^2 \sin^2(A) + c^2 - 2bc \cos(A) + b^2 \cos^2(A) \\ &= b^2(\sin^2(A) + \cos^2(A)) + c^2 - 2bc \cos(A) \end{aligned}$$

Since $\sin^2(A) + \cos^2(A) = 1$, the following statement is derived.

$$a^2 = b^2 + c^2 - 2bc \cos(A)$$

Similar reasoning produces the other components of the law of cosines.

3. The law of cosines states that $a^2 = b^2 + c^2 - 2bc \cos(A)$.

The question gives that $m\angle A = 42^\circ$, $b = 11$ m, and $c = 19$ m.

Evaluate the formula with the given information to find the approximate length of side a.

$$\begin{aligned}
 a^2 &= b^2 + c^2 - 2bc \cos(A) \\
 a^2 &= (11 \text{ m})^2 + (19 \text{ m})^2 - 2(11 \text{ m})(19 \text{ m})\cos(42^\circ) \\
 \sqrt{a^2} &\approx \sqrt{171.37 \text{ m}^2} \\
 a &\approx 13.09 \text{ m}
 \end{aligned}$$

4. The law of sines states that if ABC is a triangle with sides a, b, and c, then the following is true.

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$

The question gives that $m\angle A = 114^\circ$, $m\angle C = 22^\circ$, and $a = 50$ ft.

First, use the given information to find the measure of angle B.

$$\begin{aligned}
 m\angle A + m\angle B + m\angle C &= 180^\circ \\
 114^\circ + m\angle B + 22^\circ &= 180^\circ \\
 m\angle B &= 44^\circ
 \end{aligned}$$

Apply the law of sines to solve for the length of side b.

$$\begin{aligned}
 \frac{\sin(A)}{a} &= \frac{\sin(B)}{b} \\
 \frac{\sin(114^\circ)}{50 \text{ ft}} &= \frac{\sin(44^\circ)}{b} \\
 b &= \frac{(50 \text{ ft})\sin(44^\circ)}{\sin(114^\circ)} \\
 b &\approx 38.02 \text{ ft}
 \end{aligned}$$

5. The law of cosines states that $c^2 = a^2 + b^2 - 2ab \cos(C)$.

The question gives that $a = 7$ m, $b = 14$ m, and $c = 18$ m.

Evaluate the formula with the given information to find the approximate measure of angle C.

$$c^2 = a^2 + b^2 - 2ab \cos(C)$$

$$2ab \cos(C) = a^2 + b^2 - c^2$$

$$\cos(C) = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos(C) = \frac{(7 \text{ m})^2 + (14 \text{ m})^2 - (18 \text{ m})^2}{2(7 \text{ m})(14 \text{ m})}$$

$$\cos(C) \approx -0.4031$$

$$C \approx \cos^{-1}(-0.4031)$$

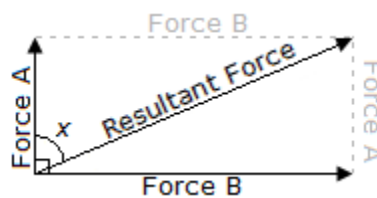
$$C \approx 113.77^\circ$$

6. The law of sines states that if ABC is a triangle with side lengths a , b , and c , the following statement is true.

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

First, sketch a picture of the forces acting on the object.

The resultant force vector is the diagonal of the rectangle formed by the force A vector and the force B vector.



Note: picture not drawn to scale

Next, use the law of sines to find the measure of the angle, x , formed by the resultant force and force A.

$$\frac{65 \text{ pounds}}{\sin(90^\circ)} = \frac{60 \text{ pounds}}{\sin(x)}$$

$$(65 \text{ pounds})\sin(x) = (60 \text{ pounds})\sin(90^\circ)$$

$$\sin(x) = \frac{(60 \text{ pounds})\sin(90^\circ)}{65 \text{ pounds}}$$

$$\sin(x) = \frac{12 \text{ pounds}}{13 \text{ pounds}}$$

$$x = \sin^{-1}\left(\frac{12}{13}\right)$$

$$x \approx 67^\circ$$

Thus, to the nearest degree, the measure of the angle formed by the resultant force and force A is **67°**.

7. The law of sines states that if ABC is a triangle with sides a , b , and c , then the following is true.

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

The question gives that $m\angle A = 49^\circ$, $a = 14$ cm, and $c = 8$ cm.

Apply the law of sines to solve for the measure of angle C.

$$\frac{a}{\sin(A)} = \frac{c}{\sin(C)}$$

$$a\sin(C) = c\sin(A)$$

$$\sin(C) = \frac{c\sin(A)}{a}$$

$$\sin(C) = \frac{(8 \text{ cm})\sin(49^\circ)}{14 \text{ cm}}$$

$$\sin(C) = 0.4313$$

$$C = \sin^{-1}(0.4313)$$

$$C \approx 25.55^\circ, 154.45^\circ$$

Notice that the calculations gave two possible angles between 0° and 180° . This is because sine is positive in both the first and second quadrants.

Since $c < a$, the measure of angle C must be less than the measure of angle A, which only occurs when the measure of angle C is **25.55°**.

8. The law of sines states that if ABC is a triangle with sides a , b , and c , then the following is true.

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

The question gives that $m\angle B = 106^\circ$, $m\angle A = 44^\circ$, and $b = 12$ in.

First, use the given information to find the measure of angle C.

$$m\angle A + m\angle B + m\angle C = 180^\circ$$

$$44^\circ + 106^\circ + m\angle C = 180^\circ$$

$$m\angle C = 30^\circ$$

Next, apply the law of sines to solve for the length of side c.

$$\frac{c}{\sin(C)} = \frac{b}{\sin(B)}$$

$$\frac{c}{\sin(30^\circ)} = \frac{12 \text{ in}}{\sin(106^\circ)}$$

$$c = \frac{(12 \text{ in})\sin(30^\circ)}{\sin(106^\circ)}$$

$$c \approx 6.24 \text{ in}$$

9. The law of cosines states that if ABC is a triangle with sides a , b , and c , the following statements are true.

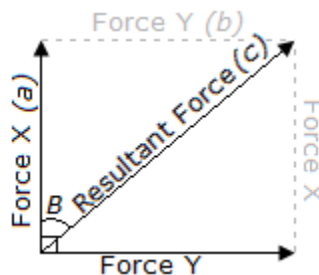
$$a^2 = b^2 + c^2 - (2bc)\cos(A)$$

$$b^2 = a^2 + c^2 - (2ac)\cos(B)$$

$$c^2 = a^2 + b^2 - (2ab)\cos(C)$$

First, sketch a picture of the forces acting on the object.

The resultant force vector is the diagonal of the rectangle formed by the force X vector and the force Y vector.



Note: Picture is not drawn to scale.

Next, use the law of cosines to find the measure of the angle, B , formed by force X and the resultant force.

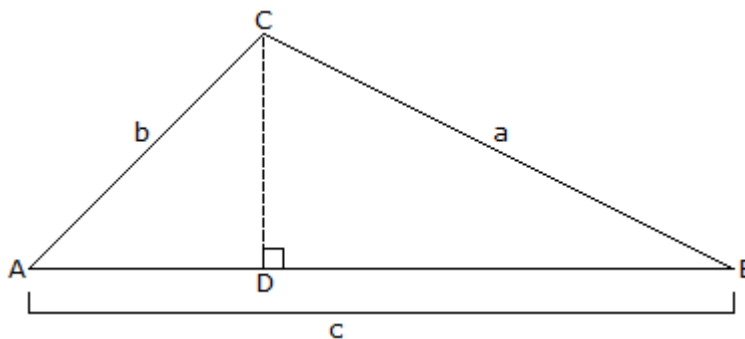
$$\begin{aligned}
 b^2 &= a^2 + c^2 - (2ac)\cos(B) \\
 (48 \text{ pounds})^2 &= (36 \text{ pounds})^2 + (60 \text{ pounds})^2 - 2(36 \text{ pounds})(60 \text{ pounds})\cos(B) \\
 2,304 \text{ pounds}^2 &= 4,896 \text{ pounds}^2 - (4,320 \text{ pounds}^2)\cos(B) \\
 (4,320 \text{ pounds}^2)\cos(B) &= 2,592 \text{ pounds}^2 \\
 \cos(B) &= 0.6 \\
 B &= \cos^{-1}(0.6) \\
 B &\approx 53^\circ
 \end{aligned}$$

Therefore, the approximate measure of the angle formed by force X and the resultant force is **53°** .

10. The law of sines states that if ABC is a triangle with sides a , b , and c , then the following is true.

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

Given triangle ABC , construct **an altitude** of the triangle. In this case, altitude CD was constructed, as shown below.



Two right triangles, ACD and BCD , were formed. From these two right triangles, the following statements can be made.

$$\begin{aligned}
 \sin(A) &= \frac{\text{opposite}}{\text{hypotenuse}} = \frac{CD}{b} \\
 \sin(B) &= \frac{\text{opposite}}{\text{hypotenuse}} = \frac{CD}{a}
 \end{aligned}$$

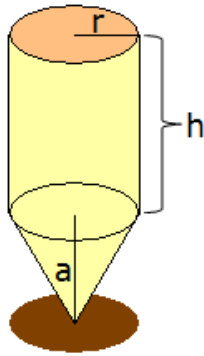
Thus, $CD = b \sin(A)$ and $CD = a \sin(B)$, which gives the following.

$$\begin{aligned}
 b \sin(A) &= a \sin(B) \\
 \frac{a}{\sin(A)} &= \frac{b}{\sin(B)}
 \end{aligned}$$

Similar reasoning produces the other components of the law of sines.

Geometry: Volume

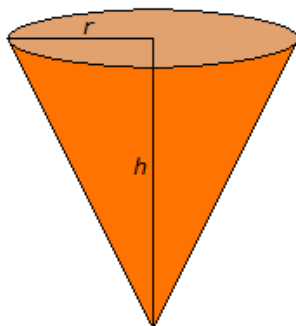
1. Pamela made a bird feeder with a can and a funnel, as shown below.



If $r = 4$ cm, $h = 18$ cm, and $a = 9$ cm, what is the volume of birdseed that the bird feeder can hold at full capacity?

- ☐ A. 432π cm^3
- ☐ B. 144π cm^3
- ☐ C. 336π cm^3
- ☐ D. 384π cm^3

2.



Note: Figure not drawn to scale.

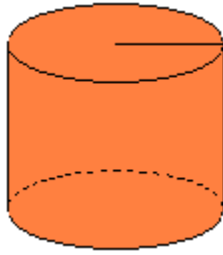
If the height of the cone, h , is 6 centimeters and the radius of the base, r , is 2 centimeters, what is the volume of the cone?

- ☐ A. 36π centimeters³
 - ☐ B. 4π centimeters³
 - ☐ C. 24π centimeters³
 - ☐ D. 8π centimeters³
-

3. At the store, Jason finds the total volume of soda in a pack is 141.3 cubic inches. If each cylinder-shaped can has a height of 4 inches and a diameter of 3 inches, how many soda cans are in a pack? (Use 3.14 for π).

- ☐ A. 5
- ☐ B. 6
- ☐ C. 3
- ☐ D. 28

4.

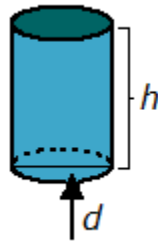


Note: Figure is not drawn to scale.

If the radius of the base is 6 units and the height is 9 units, what is the volume of the cylinder shown above?

- ☐ A. 324π cubic units
 - ☐ B. 144π cubic units
 - ☐ C. 216π cubic units
 - ☐ D. 180π cubic units
-

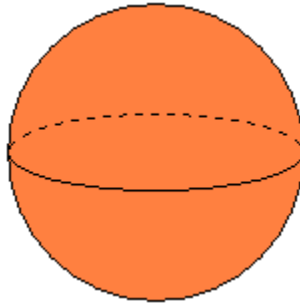
5. In the cylinder below, the diameter, d , is 12 in, and the altitude, h , is 7 in. What is the volume of the cylinder?



**picture not drawn to scale*

- ☐ A. 120π in³
- ☐ B. 156π in³
- ☐ C. 252π in³
- ☐ D. 432π in³

6.



If the volume of the sphere is 36π units³, what is the radius of the sphere?

- ☐ A. 6 units
 - ☐ B. 3 units
 - ☐ C. 1 unit
 - ☐ D. 12 units
-

7. A souvenir shop sells a liquid-filled, triangular-based pyramid paperweight. The height of the base is 3 centimeters and the base length is twice the base height. If the height of the pyramid is 6 centimeters, what is its volume to the nearest cubic centimeter?

- ☐ A. 36 cm³
 - ☐ B. 18 cm³
 - ☐ C. 9 cm³
 - ☐ D. 49 cm³
-

8. A mug can hold 32.04 ounces of coffee without overflowing. The radius of the mug is 5 centimeters. Given that 1 cubic centimeter equals 0.034 ounce, what is the height of the mug to the nearest centimeter?

- ☐ A. 10 cm
- ☐ B. 17 cm
- ☐ C. 14 cm
- ☐ D. 12 cm

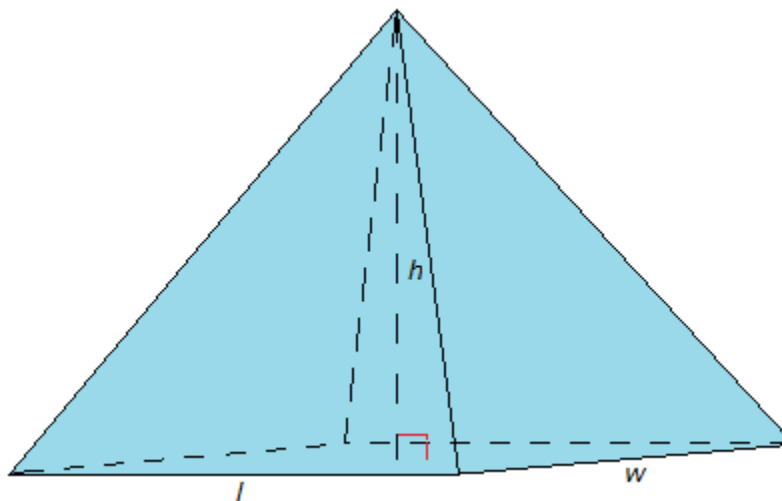
9. The sphere at the top of a water tower holds enough water to pressurize the water supply for the city. The radius of the sphere measures 12 yards.

Throughout the day, the water tower lost 2,143.57 cubic yards of water. If the water tower was completely full at the beginning of the day, about how much water is left in the water tower at the end of the day? (Use 3.14 for π .)

$$\text{Volume of a Sphere} = \frac{4}{3}\pi r^3$$

- ☐ A. 5,091 cubic yards
 - ☐ B. 1,005 cubic yards
 - ☐ C. 2,144 cubic yards
 - ☐ D. 7,235 cubic yards
-

10. In the rectangular pyramid below, the length, l , is 6 in, the width, w , is 2 in, and the height, h , is 16 in. What is the volume of the pyramid?



Note: Figure not drawn to scale.

- ☐ A. $21\frac{1}{3} \text{ in}^3$
- ☐ B. 192 in^3
- ☐ C. 64 in^3
- ☐ D. 512 in^3

Answers: Volume

1. C
2. D
3. A
4. A
5. C
6. B
7. B
8. D
9. A
10. C

Explanations

1. First, find the volume of the cylinder-shaped portion of the bird feeder.

$$\begin{aligned}V_{\text{cyl}} &= \pi r^2 h \\&= \pi (4 \text{ cm})^2 (18 \text{ cm}) \\&= \pi (16 \text{ cm}^2) (18 \text{ cm}) \\&= 288\pi \text{ cm}^3\end{aligned}$$

Next, find the volume of the cone-shaped portion of the bird feeder.

$$\begin{aligned}V_{\text{cone}} &= \frac{1}{3}\pi r^2 a \\&= \frac{1}{3}\pi (4 \text{ cm})^2 (9 \text{ cm}) \\&= \frac{1}{3}\pi (16 \text{ cm}^2) (9 \text{ cm}) \\&= 48\pi \text{ cm}^3\end{aligned}$$

Last, find the volume of birdseed that the bird feeder can hold at full capacity.

$$\begin{aligned}V_{\text{full}} &= V_{\text{cyl}} + V_{\text{cone}} \\&= 288\pi \text{ cm}^3 + 48\pi \text{ cm}^3 \\&= 336\pi \text{ cm}^3\end{aligned}$$

2. Multiply $\frac{1}{3}$ times the area of the base times the height to find the volume of the cone.

$$\begin{aligned}
 \text{Volume} &= \frac{1}{3}\pi r^2 h \\
 &= \frac{1}{3}\pi(2 \text{ centimeters})^2(6 \text{ centimeters}) \\
 &= \frac{1}{3}(24\pi \text{ centimeters}^3) \\
 &= 8\pi \text{ centimeters}^3
 \end{aligned}$$

3. Use the formula for the volume of a cylinder to find the volume of a can. Remember to divide the length of the diameter by two to find the length of the radius. Use 3.14 for π .

$$\begin{aligned}
 V &= \pi r^2 h \\
 V &= 3.14(1.5 \text{ in})^2(4 \text{ in}) \\
 V &= 28.26 \text{ in}^3
 \end{aligned}$$

Then, find the number of cans by dividing the total volume of soda by the volume of each can.

$$141.3 \text{ in}^3 \div 28.26 \text{ in}^3 \text{ per can} \approx 5_{\text{cans}}$$

Each pack contains **5** soda cans.

4. To find the volume of the cylinder, multiply the area of the base and the height of the cylinder.

$$\begin{aligned}
 \text{Volume} &= \pi r^2 h \\
 &= \pi(6 \text{ units})^2(9 \text{ units}) \\
 &= 324\pi \text{ cubic units}
 \end{aligned}$$

5. Use the formula for the volume of a cylinder.

$$\text{Volume} = Bh$$

The base is a circle. Therefore, the area of the base is $B = \pi r^2$ and the volume formula becomes $V = \pi r^2 h$.

Use the given values to find the volume of the cylinder. Since the diameter is 12 in, the radius is 6 in.

$$\begin{aligned}
 V &= \pi r^2 h \\
 &= \pi(6 \text{ in})^2 \cdot (7 \text{ in}) \\
 &= \pi(36 \text{ in}^2) \cdot (7 \text{ in}) \\
 &= 252\pi \text{ in}^3
 \end{aligned}$$

6. Substitute $V = 36\pi \text{ units}^3$ into the formula for the volume of a sphere and solve for r .

$$\begin{aligned}
 V &= \frac{4}{3}\pi r^3 \\
 36\pi \text{ units}^3 &= \frac{4}{3}\pi r^3 \\
 27 \text{ units}^3 &= r^3 \\
 3 \text{ units} &= r
 \end{aligned}$$

So, the radius of the sphere is **3 units**.

7. The volume of a pyramid can be found using the formula below, where B is the area of the base and h is the altitude. The base of the pyramid is a triangle.

$$\begin{aligned}
 V &= \frac{1}{3}Bh \\
 &= \frac{1}{3}\left(\frac{1}{2}bh_{\text{base}}\right)h_{\text{pyramid}} \\
 &= \frac{1}{6}(b)(h_{\text{base}})(h_{\text{pyramid}}) \\
 &= \frac{1}{6}(2 \cdot 3 \text{ cm})(3 \text{ cm})(6 \text{ cm}) \\
 &= 18 \text{ cm}^3
 \end{aligned}$$

Therefore, the volume of the paperweight is **18 cm³**.

8. Convert the volume of the coffee mug from ounces to cubic centimeters.

$$32.04 \text{ oz} \cdot \frac{1 \text{ cm}^3}{0.034 \text{ oz}} \approx 942.35 \text{ cm}^3$$

Use the formula for the volume of a cylinder, where B is the area of the base and h is the altitude, to solve for the height of the mug. The base of a cylinder is a circle.

$$\begin{aligned}
 V &= Bh \\
 V &= (\pi r^2)h \\
 942.35 \text{ cm}^3 &\approx \pi(5 \text{ cm})^2h \\
 \frac{942.35 \text{ cm}^3}{25\pi \text{ cm}^2} &\approx h \\
 12 \text{ cm} &\approx h
 \end{aligned}$$

Therefore, the approximate height of the mug is **12 cm**.

9. Find the volume of the water in the sphere. Use 3.14 for π .

$$\begin{aligned}
 V &= \frac{4}{3}\pi r^3 \\
 &= \frac{4}{3}(3.14)(12 \text{ yd})^3 \\
 &\approx 7,234.56 \text{ yd}^3
 \end{aligned}$$

To find how much water remains in the sphere at the end of the day, subtract the amount of water lost from the total volume of the sphere.

$$7,234.56 \text{ yd}^3 - 2,143.57 \text{ yd}^3 \approx 5091 \text{ yd}^3$$

At the end of the day described, the sphere contains **5,091 cubic yards** of water.

10. To find the volume of a pyramid, use the following formula.

$$V = \frac{1}{3}Bh$$

In this case, the base is a rectangle; therefore, the area of the base is given below.

$$B = lw$$

Thus, the volume formula becomes the following.

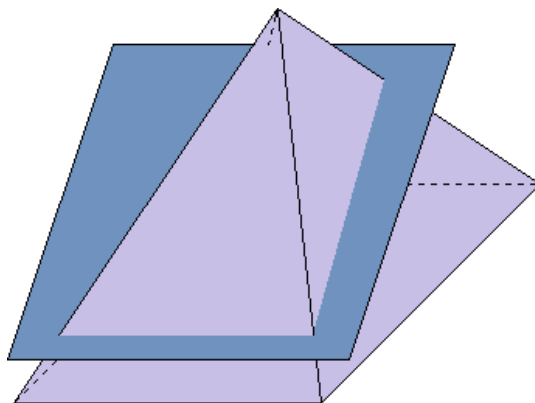
$$V = \frac{1}{3}lwh$$

Use the given values to find the volume of the pyramid.

$$\begin{aligned}
 V &= \frac{1}{3}lwh \\
 &= \frac{1}{3}(6 \text{ in})(2 \text{ in})(16 \text{ in}) \\
 &= \frac{1}{3}(192 \text{ in}^3) \\
 &= 64 \text{ in}^3
 \end{aligned}$$

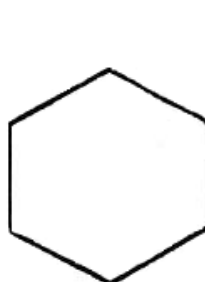
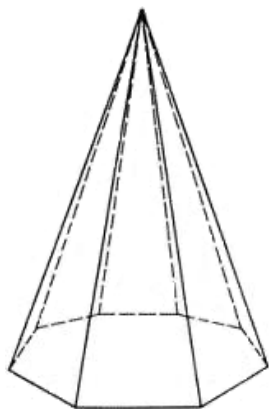
Geometry: Two- and Three-Dimensional Objects

1. A rectangular pyramid is sliced diagonally, with a plane that passes through the edges of the base, as shown below. Which figure best represents the cross-section parallel to the slice?

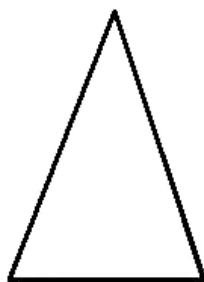


- ☐ A. triangle
 - ☐ B. rectangle
 - ☐ C. square
 - ☐ D. trapezoid
-

2. An octagonal-based pyramid is shown below. If the figure is sliced horizontally, which image best represents the cross-section parallel to the slice?



W.



X.



Y.



Z.

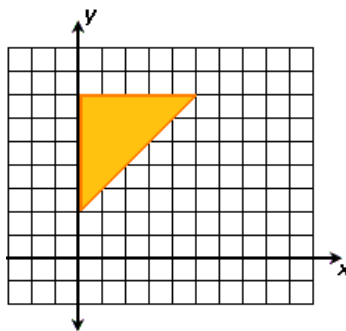
- ☐ A. Z
- ☐ B. X
- ☐ C. Y
- ☐ D. W

3. Marta is designing a new cone to be used by the department of transportation. In order to design a cone that will withstand high winds, she is planning to base the design on a 30-60-90 triangle. After determining the correct side lengths for the cone, she will create the cone by rotating the triangle along its vertical side.

If the base angle of the cone needs to measure 60° and the cone needs to be 24 inches tall, approximately how wide will the base of the resulting cone need to be?

- ☐ A. 83.14 inches
- ☐ B. 41.57 inches
- ☐ C. 13.86 inches
- ☐ D. 27.72 inches

4.



If the image above is rotated about the x-axis, which of the following images best represents the result?



W.



X.



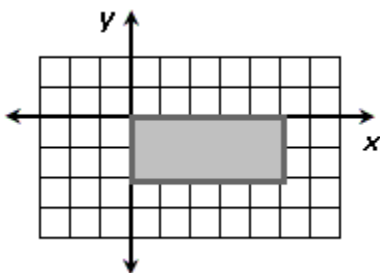
Y.



Z.

- ☐ A. Z
- ☐ B. Y
- ☐ C. X
- ☐ D. W

5.



If the image above is rotated about the x-axis, which of the following images best represents the result?



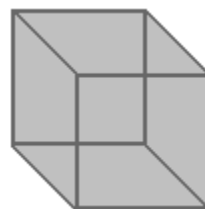
W.



X.



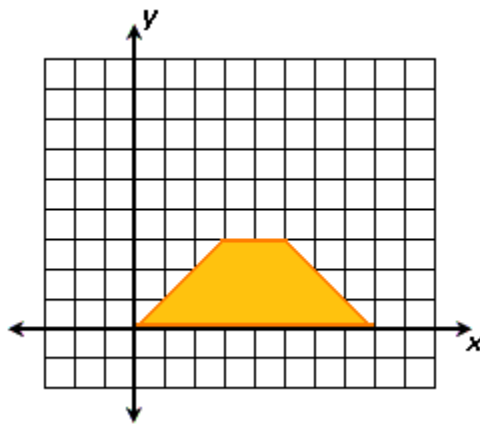
Y.



Z.

- ☐ A. Y
- ☐ B. Z
- ☐ C. W
- ☐ D. X

6.



If the image above is rotated about the y -axis, which of the following images best represents the result?



W.



X.



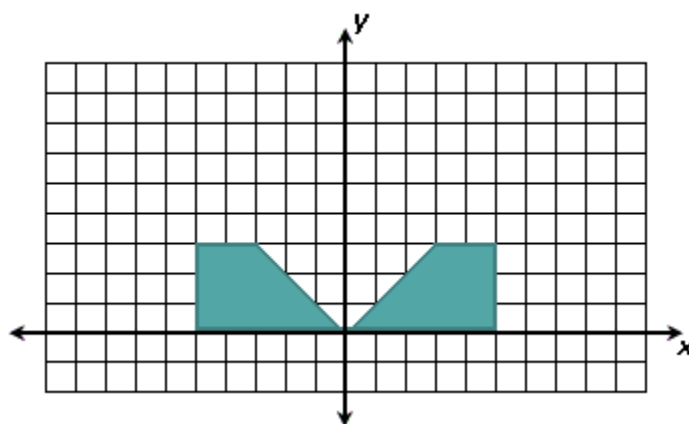
Y.



Z.

- ☐ A. X
- ☐ B. Z
- ☐ C. W
- ☐ D. Y

7.



If the image above is rotated about the x -axis, which of the following images best represents the result?



W.



X.



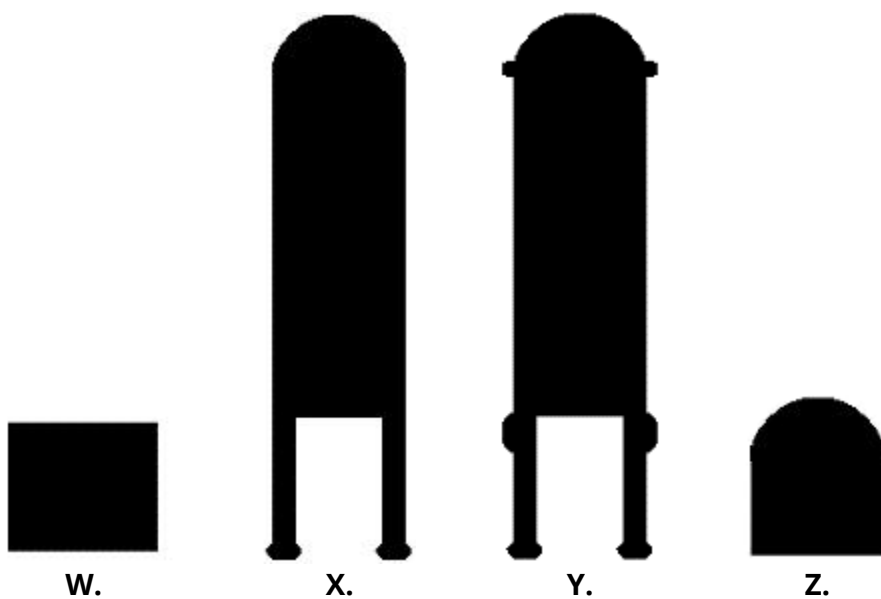
Y.



Z.

- ☐ A. X
- ☐ B. W
- ☐ C. Y
- ☐ D. Z

8. An image of a display cabinet is shown below. The display area is a rectangular prism. If the cabinet is sliced horizontally near the center, which image best represents the cross-section parallel to the slice?



- ☐ A. Z
- ☐ B. X
- ☐ C. Y
- ☐ D. W

9. An image of a tennis trophy is shown below. The base is a rectangular prism. The top piece is spherical, tangential to the base, and centered atop the base. If the trophy is sliced vertically top to bottom, which image best represents the cross-section parallel to the slice?



W.



X.



Y.



Z.

- ☐ A. Z
 - ☐ B. X
 - ☐ C. W
 - ☐ D. Y
-

10. Jake is planning to build a ramp for the entrance into his building so that people in wheelchairs will have easier access. The entrance to the building is 3 feet above the ground. He wants the angle of elevation to be 5° .

How far away from the base of the building does the ramp need to begin?

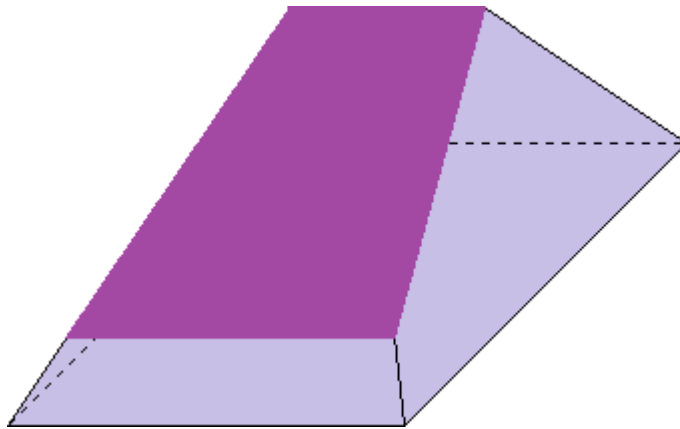
- ☐ A. 3.01 feet
- ☐ B. 34.29 feet
- ☐ C. 34.42 feet
- ☐ D. 0.26 feet

Answers: Two-and -Three-Dimensional Objects

1. D
2. C
3. D
4. C
5. D
6. C
7. A
8. D
9. C
10. B

Explanations

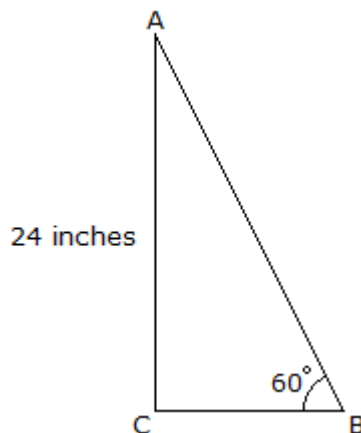
1. The resulting cross-section is a **trapezoid**.



2. Slicing an octagonal-based pyramid horizontal to the base produces a cross-sectional image which is similar to the base.

Therefore, the correct image is **Y**.

3. Start by creating a diagram of the triangle that is described.



Next, find the length of the base of the triangle, which is represented by side BC, by setting up an equation using the values given in the question.

$$\tan(60^\circ) = \frac{24 \text{ inches}}{BC}$$

$$\sqrt{3} = \frac{24 \text{ inches}}{BC}$$

$$BC = \frac{24 \text{ inches}}{\sqrt{3}}$$

$$BC \approx 13.86 \text{ inches}$$

When the triangle is rotated about the line segment AC, the length of the base of the resulting cone will be twice the length of segment BC.

$$13.86 \text{ inches} \times 2 = 27.72 \text{ inches}$$

Therefore, the base of the resulting cone will be approximately **27.72 inches** wide.

4. Rotating this image about the x-axis will result in a cylinder with a disc-like hollow.

Rotating this image about the y-axis will result in a cone.

The correct image is **X**.

5. Rotating this image about either axis will result in a cylinder.

Rotating it about the y-axis will result in a shorter, wider cylinder than rotating it about the x-axis.

The correct image is **X**.

6. Rotating this image about the x-axis will result in a cylinder with a cone top and a cone bottom.

Rotating this image about the y-axis will result in a disc-like shape with an angled rim and a conical hollow.

The correct image is **W**.

7. Rotating this image about the x-axis will result in two separate shapes with a cylinder bottom and cone top that meet at the vertices of the cones.

Rotating this image about the y-axis will result in a cylinder with a conical hollow.

The correct image is **X**.

8. Images Y and X are views corresponding to vertical slices.

The rectangular portion of image Z corresponds to a horizontal slice; however, it also has a semi-circle portion which would not appear in the horizontal slice.

Slicing a rectangular prism horizontal to the base produces a cross-sectional image which is the same as the base.

Therefore, the correct image is **W**.

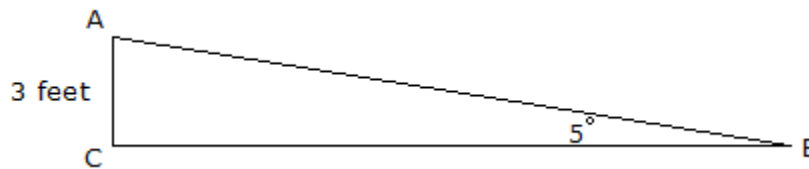
9. Image Z best represents a horizontal slice through the base of the trophy.

Image Y best represents a vertical slice through the base of the trophy, but it does not include the top piece.

Image X does take into account the tangential placement of the top piece of the trophy.

The correct image of the cross-section is **W**.

10. Ramps are built in the shape of a right rectangular prism. Start by drawing a diagram of the right triangular cross-section.



The distance from the base of the building is represented by the length of line segment BC. Set up an equation using the given values to find the length of line segment BC.

$$\tan(5^\circ) = \frac{3 \text{ feet}}{BC}$$

$$BC = \frac{3 \text{ feet}}{\tan(5^\circ)}$$

$$BC \approx 34.29 \text{ feet}$$

Therefore, the ramp will begin approximately **34.29 feet** from the base of the building.

Geometry: Coordinate Geometry

1. Ethan wants to purchase a rectangular lot at a lake resort. He drew the layout of the lot on a coordinate plane, where the x - and y -values represent the length, in yards.

The center of the lot is located at $(0, 0)$.

The northeast corner of the lot is located at $(16, 13)$.

The southeast corner of the lot is located at $(16, -13)$.

The northwest corner of the lot is located at $(-16, 13)$.

The southwest corner of the lot is located at $(-16, -13)$.

What is the area of the rectangular lot?

- ☐ A. 416 square yards
 - ☐ B. 1,664 square yards
 - ☐ C. 116 square yards
 - ☐ D. 832 square yards
-

2. The vertices of a triangle are listed below.

$$Q(-7,7), R(6,7), S(-9,-3)$$

Which of the following correctly classifies the triangle?

- ☐ A. The triangle is a right scalene triangle.
 - ☐ B. The triangle is a right isosceles triangle.
 - ☐ C. The triangle is an acute scalene triangle.
 - ☐ D. The triangle is an obtuse scalene triangle.
-

3. The vertices of rectangle WXYZ are listed below.

$$W(4,9), X(7,7), Y(1,-2), Z(-2,0)$$

What is the approximate perimeter of rectangle WXYZ?

- ☐ A. 28.84 units
- ☐ B. 43.27 units
- ☐ C. 14.42 units
- ☐ D. 39 units

4. The vertices of quadrilateral WXYZ are listed below.

$$W(6,-3), X(12,-3), Y(16,-9), Z(6,-9)$$

What is the approximate perimeter of the quadrilateral?

- ☐ A. 23.21 units
 - ☐ B. 43.27 units
 - ☐ C. 29.21 units
 - ☐ D. 14.61 units
-

5. Jason is camping at a state park and must set up his tent in a given rectangular section. His tent has five corners that must be secured into the ground with stakes. Suppose the locations of the stakes are plotted on a coordinate plane, where the x - and y -values represent the position, in inches, from the southwest corner of his rectangular section, which is located at $(0, 0)$. The locations of the stakes are as follows.

Stake J is located at $(24, 72)$.

Stake K is located at $(72, 120)$.

Stake L is located at $(120, 72)$.

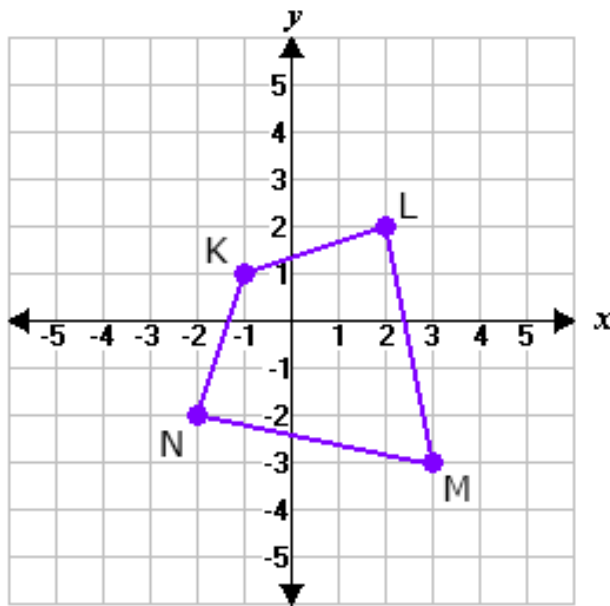
Stake M is located at $(96, 24)$.

Stake N is located at $(48, 24)$.

What is the approximate perimeter of the base of his tent?

- ☐ A. 291 inches
- ☐ B. 237 inches
- ☐ C. 192 inches
- ☐ D. 994 inches

6. What is the most specific name for figure KLMN?



- ☐ A. parallelogram
- ☐ B. rectangle
- ☐ C. kite
- ☐ D. quadrilateral

7. The vertices of triangle RST are listed below.

$$R(-6,4), S(18,4), T(6,-1)$$

What is the perimeter of triangle *RST*?

- ☐ A. 50 units
- ☐ B. 78 units
- ☐ C. 120 units
- ☐ D. 37 units

8. The vertices of a rectangle are listed below.

$$I(5, 8), J(5, -2), K(-6, -2), L(-6, 8)$$

What is the area of the rectangle?

- ☐ A. 42 square units
- ☐ B. 440 square units
- ☐ C. 110 square units
- ☐ D. 220 square units

9. The vertices of a triangle are listed below.

$$R(1, 5), S(4, -4), T(-5, -4)$$

What is the area of the triangle?

- ☐ A. 20.25 square units
 - ☐ B. 81 square units
 - ☐ C. 29.3 square units
 - ☐ D. 40.5 square units
-

10. The vertices of a quadrilateral are listed below.

$$W(8, 7), X(10, 5), Y(4, -1), Z(2, 1)$$

Which of the following is the strongest classification that identifies this quadrilateral?

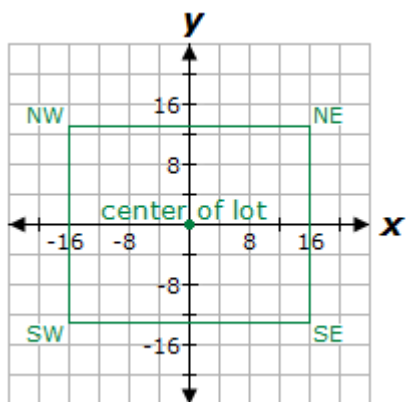
- ☐ A. The quadrilateral is a rectangle.
- ☐ B. The quadrilateral is a square.
- ☐ C. The quadrilateral is a trapezoid.
- ☐ D. The quadrilateral is a rhombus.

Answers: Coordinate Geometry

1. D
2. D
3. A
4. C
5. A
6. C
7. A
8. C
9. D
10. A

Explanations

1. First, sketch a picture of the lot on a coordinate plane.



Next, use the distance formula to find the length of two adjacent sides of the rectangular lot.

Side	Length Calculation	Length
<i>NW to NE</i>	$\sqrt{(-16 - 16)^2 + (13 - 13)^2}$	$\sqrt{1,024} = 32$
<i>NE to SE</i>	$\sqrt{(16 - 16)^2 + (13 - (-13))^2}$	$\sqrt{676} = 26$

Then, find the area of the rectangular lot.

$$\begin{aligned}
 A &= (\text{length})(\text{width}) \\
 &= (32 \text{ yards})(26 \text{ yards}) \\
 &= 832 \text{ square yards}
 \end{aligned}$$

Therefore, the area of the rectangular lot is **832 square yards**.

2. Use the distance formula to find the length of each side of the triangle.

Side	Length Calculation	Length	Length Squared
QR	$\sqrt{(-7 - 6)^2 + (7 - 7)^2}$	$\sqrt{169} = 13$	169
RS	$\sqrt{(6 - (-9))^2 + (7 - (-3))^2}$	$\sqrt{325} \approx 18.03$	325
SQ	$\sqrt{(-9 - (-7))^2 + (-3 - 7)^2}$	$\sqrt{104} \approx 10.2$	104

Since the three sides all have different lengths, the triangle is scalene.

Where a , b , and c are the lengths of the sides and c is the longest side,

if $c^2 < a^2 + b^2$, then it is an acute triangle,

if $c^2 = a^2 + b^2$, then it is a right triangle, and

if $c^2 > a^2 + b^2$, then it is an obtuse triangle.

Since the square of the longest side is greater than the sum of the squares of the other two sides, the triangle is obtuse.

Therefore, **the triangle is an obtuse scalene triangle**.

3. The perimeter of a rectangle is equal to the sum of the lengths of the sides of the rectangle.

First, use the distance formula to find the length of each side of the rectangle.

Side	Length Calculation	Length
WX	$\sqrt{(4 - 7)^2 + (9 - 7)^2}$	$\sqrt{13}$
XY	$\sqrt{(7 - 1)^2 + (7 - (-2))^2}$	$\sqrt{117} = 3\sqrt{13}$
YZ	$\sqrt{(1 - (-2))^2 + (-2 - 0)^2}$	$\sqrt{13}$
ZW	$\sqrt{(-2 - 4)^2 + (0 - 9)^2}$	$\sqrt{117} = 3\sqrt{13}$

Then, add the lengths of the sides of the rectangle to find the approximate perimeter.

$$\begin{aligned}
 \text{Perimeter} &= WX + XY + YZ + ZW \\
 &= \sqrt{13} \text{ units} + 3\sqrt{13} \text{ units} + \sqrt{13} \text{ units} + 3\sqrt{13} \text{ units} \\
 &\approx 28.84 \text{ units}
 \end{aligned}$$

4. First, find the length of the sides of the quadrilateral by using the distance formula.

Side Length Calculation **Length**

WX $\sqrt{(6 - 12)^2 + (-3 - (-3))^2} \sqrt{36}$

XY $\sqrt{(12 - 16)^2 + (-3 - (-9))^2} \sqrt{52}$

YZ $\sqrt{(16 - 6)^2 + (-9 - (-9))^2} \sqrt{100}$

ZW $\sqrt{(6 - 6)^2 + (-9 - (-3))^2} \sqrt{36}$

Next, to find the perimeter of the quadrilateral, calculate the sum of the side lengths.

$$\sqrt{36} \text{ units} + \sqrt{52} \text{ units} + \sqrt{100} \text{ units} + \sqrt{36} \text{ units} \approx 29.21 \text{ units}$$

Therefore, the perimeter of the quadrilateral is approximately **29.21 units**.

5. First, calculate the length, in inches, of each side of the tent using the distance formula.

Side	Length Calculation	Length
JK	$\sqrt{(72 - 24)^2 + (120 - 72)^2}$	$\sqrt{4,608} = 48\sqrt{2}$
KL	$\sqrt{(120 - 72)^2 + (72 - 120)^2}$	$\sqrt{4,608} = 48\sqrt{2}$
LM	$\sqrt{(96 - 120)^2 + (24 - 72)^2}$	$\sqrt{2,880} = 24\sqrt{5}$
MN	$\sqrt{(48 - 96)^2 + (24 - 24)^2}$	$\sqrt{2,304} = 48$
NJ	$\sqrt{(24 - 48)^2 + (72 - 24)^2}$	$\sqrt{2,880} = 24\sqrt{5}$

Then, find the perimeter.

$$\text{Perimeter} = JK + KL + LM + MN + NJ$$

$$= 48\sqrt{2} \text{ inches} + 48\sqrt{2} \text{ inches} + 24\sqrt{5} \text{ inches} + 48 \text{ inches} + 24\sqrt{5} \text{ inches}$$

$$\approx 291 \text{ inches}$$

Therefore, the perimeter of the base of the tent is approximately **291 inches**.

6.

$$\text{distance} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Use the distance formula shown above and the coordinates of the points to determine the length of each side of the figure.

$$\begin{aligned}\text{length of KL} &= \sqrt{(-1 - 2)^2 + (1 - 2)^2} \\ &= \sqrt{(-3)^2 + (-1)^2} \\ &= \sqrt{10}\end{aligned}$$

$$\begin{aligned}\text{length of LM} &= \sqrt{(2 - 3)^2 + (2 - (-3))^2} \\ &= \sqrt{(1)^2 + (5)^2} \\ &= \sqrt{26}\end{aligned}$$

$$\begin{aligned}\text{length of MN} &= \sqrt{(3 - (-2))^2 + (-3 - (-2))^2} \\ &= \sqrt{(5)^2 + (-1)^2} \\ &= \sqrt{26}\end{aligned}$$

$$\begin{aligned}\text{length of NK} &= \sqrt{(-2 - (-1))^2 + (-2 - 1)^2} \\ &= \sqrt{(-1)^2 + (-3)^2} \\ &= \sqrt{10}\end{aligned}$$

The slopes of the sides can be determined by using the formula, $\frac{y_2 - y_1}{x_2 - x_1}$.

$$\text{slope of KL} = \frac{1}{3}$$

$$\text{slope of LM} = -\frac{5}{1}$$

$$\text{slope of MN} = -\frac{1}{5}$$

$$\text{slope of NK} = \frac{3}{1}$$

Since the figure has two sets of congruent, adjacent sides with reciprocal slopes, the most specific name for this shape is a **kite**.

7. The perimeter of a triangle is equal to the sum of the lengths of the sides of the triangle.

First, use the distance formula to find the length of each side of the triangle.

Side	Length Calculation	Length
RS	$\sqrt{(-6 - 18)^2 + (4 - 4)^2}$	$\sqrt{576} = 24$
ST	$\sqrt{(18 - 6)^2 + (4 - (-1))^2}$	$\sqrt{169} = 13$

$$TR = \sqrt{(6 - (-6))^2 + (-1 - 4)^2} = \sqrt{169} = 13$$

Then, add the lengths of the sides of the triangle to find the perimeter.

$$\begin{aligned} \text{Perimeter} &= RS + ST + TR \\ &= 24 \text{ units} + 13 \text{ units} + 13 \text{ units} \\ &= 50 \text{ units} \end{aligned}$$

8. First, use the distance formula to find the length of two adjacent sides of the rectangle.

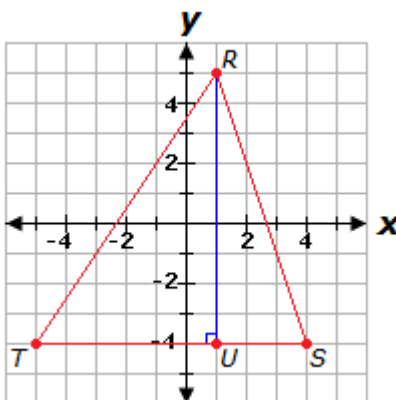
Side	Length Calculation	Length
IJ	$\sqrt{(5 - 5)^2 + (8 - (-2))^2}$	$\sqrt{100} = 10$
JK	$\sqrt{(5 - (-6))^2 + (-2 - (-2))^2}$	$\sqrt{121} = 11$

Next, find the area of the rectangle.

$$\begin{aligned} A &= (\text{length})(\text{width}) \\ &= (11 \text{ units})(10 \text{ units}) \\ &= 110 \text{ square units} \end{aligned}$$

Therefore, the area of the rectangle is **110 square units**.

9. First, graph triangle RST .



The height of triangle RST is the perpendicular distance from point R to ST . The height is represented by RU on the graph above.

Next, use the distance formula to find the length of the base, ST , and the height, RU , of the triangle.

	Length Calculation	Length
Base ST	$\sqrt{(4 - (-5))^2 + (-4 - (-4))^2}$	$\sqrt{81} = 9$
Height RU	$\sqrt{(1 - 1)^2 + (5 - (-4))^2}$	$\sqrt{81} = 9$

Then, calculate the area of the triangle.

$$\begin{aligned}
 A &= \frac{1}{2}(\text{base})(\text{height}) \\
 &= \frac{1}{2}(9 \text{ units})(9 \text{ units}) \\
 &= 40.5 \text{ square units}
 \end{aligned}$$

Therefore, the area of the triangle is **40.5 square units**.

10. Find the slope of each side of the quadrilateral.

Side	WX	XY	YZ	ZW
Slope Calc.	$\frac{5 - 7}{10 - 8}$	$\frac{-1 - 5}{4 - 10}$	$\frac{1 - (-1)}{2 - 4}$	$\frac{7 - 1}{8 - 2}$
Slope	-1	1	-1	1

Since opposite sides have equal slopes, the opposite sides are parallel. So, the shape is not a trapezoid.

Since adjacent sides have slopes that are negative reciprocals, the sides are perpendicular. So, the shape is either a rectangle or a square.

Find the length of two adjacent sides.

Side	WX	XY
Length Calc.	$\sqrt{(8 - 10)^2 + (7 - 5)^2}$	$\sqrt{(10 - 4)^2 + (5 - (-1))^2}$
Length	$\sqrt{8} \approx 2.83$	$\sqrt{72} \approx 8.49$

Since the two adjacent sides are not equal, the shape is not a square.

Therefore, **the quadrilateral is a rectangle**.

Geometry: Probability

1. Sam's closet contains blue and green shirts. He has eight blue shirts, and seven green shirts. Five of the blue shirts have stripes, and four of the green shirts have stripes. What is the probability that Sam randomly chooses a shirt that is blue or has stripes?

- ☐ A. $P(B \cup S) = 0.8$
 - ☐ B. $P(B \cup S) = 0.27$
 - ☐ C. $P(B \cup S) = 1.46$
 - ☐ D. $P(B \cup S) = 0.33$
-

2.

Peter knows that the probability of randomly choosing a comedy movie from his movie collection is $\frac{1}{2}$. If Peter randomly chooses a movie from his movie collection, what is the probability that it will not be a comedy?

- ☐ A. $\frac{1}{3}$
 - ☐ B. $\frac{2}{3}$
 - ☐ C. $\frac{1}{2}$
 - ☐ D. $\frac{5}{6}$
-

3. Bobby is taking a multiple-choice history test. He has decided to randomly guess on the first two questions. On each question there are 4 answer choices. What is the probability that he answers the first question incorrectly and the second question incorrectly?

- ☐ A. $\frac{2}{3}$
- ☐ B. $\frac{3}{16}$
- ☐ C. $\frac{1}{16}$
- ☐ D. $\frac{9}{16}$

4. A cellular company has 851 customers. The company has sold 534 talk and text plans, and they have sold 768 talk plans. What is the probability that a customer has a text plan given that they have purchased a talk plan?

- ☐ A. $P(X|T) = 0.3$
 - ☐ B. $P(X|T) = 0.7$
 - ☐ C. $P(X|T) = 1.43$
 - ☐ D. $P(X|T) = 0.63$
-

5. The following balls are placed in an urn: 4 red, 3 yellow, 6 blue, and 2 green. One ball is randomly drawn from the urn. What is the probability that the ball is either yellow or green?

- ☐ A. $\frac{1}{6}$
 - ☐ B. $\frac{1}{3}$
 - ☐ C. $\frac{1}{15}$
 - ☐ D. $\frac{2}{15}$
-

6. Ryan's mp3 player has 34 songs on it. There are 12 country songs, 10 rock songs, and 12 jazz songs. If Ryan randomly selects 2 songs, without replacement, what is the probability that one song will be country and one song will be rock?

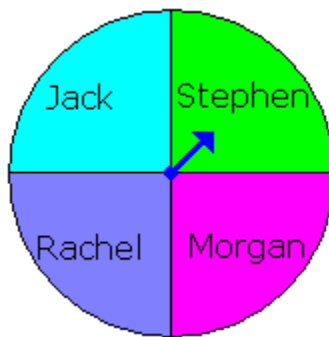
- ☐ A. $P(C \text{ and } R) = 0.9$
- ☐ B. $P(C \text{ and } R) = 0.1$
- ☐ C. $P(C \text{ and } R) = 0.38$
- ☐ D. $P(C \text{ and } R) = 0.11$

7.

Twenty-five students at Hunley Elementary made the Honor Roll. As a reward, the Principal is sending one of its Honor Roll students to Space Camp. Each student's name is placed on a piece of paper then placed in a hat. One name is randomly drawn from the hat. If there are 16 boys on the Honor Roll, what is the probability that the name drawn is a girl's?

- ☐ A. $\frac{16}{25}$
 - ☐ B. $\frac{25}{16}$
 - ☐ C. $\frac{25}{9}$
 - ☐ D. $\frac{9}{25}$
-

8. Jack and his three siblings made a spinner to decide who has to do each chore.



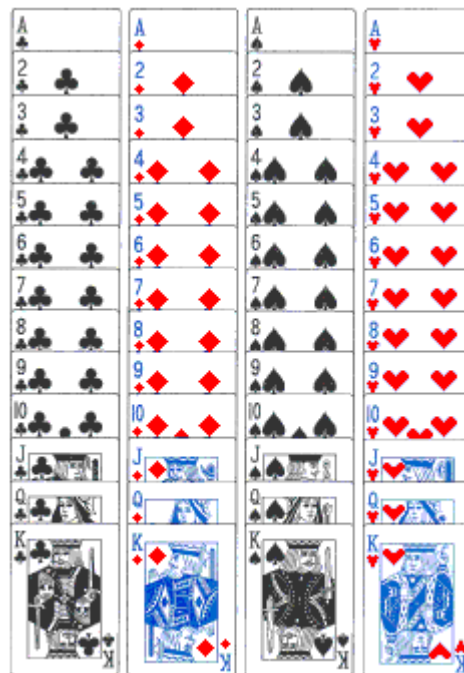
If there are two chores to do, what is the probability that Jack will have to do at least one of them?

- ☐ A. $\frac{1}{4}$
- ☐ B. $\frac{37}{64}$
- ☐ C. $\frac{3}{4}$
- ☐ D. $\frac{7}{16}$

9. There are five times as many whitetail deer as mule deer in the woods. If Jacob gets one deer in the woods, what is the probability that Jacob gets a mule deer?

- ☐ A. $\frac{1}{7}$
- ☐ B. $\frac{1}{5}$
- ☐ C. $\frac{1}{6}$
- ☐ D. $\frac{5}{6}$

10. A standard deck of 52 playing cards has 4 suits. They are clubs (\clubsuit), diamonds (\diamondsuit), spades (\spadesuit), and hearts (\heartsuit). The deck also has 13 different ranks. They are Ace (A), 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack (J), Queen (Q), and King (K). Johnny draws 5 cards from a standard deck of 52 cards. If the cards are well shuffled, what is the probability of Johnny drawing 5 cards where 3 are one rank and 2 are another rank? (example: a 5 of clubs, a 5 of spades, a 5 of diamonds, a King of hearts, and a King of diamonds)



- ☐ A. $\frac{1}{108,290}$
- ☐ B. $\frac{1}{199,920}$
- ☐ C. $\frac{1}{16,660}$
- ☐ D. $\frac{6}{4,165}$

Answers: Probability

1. A
2. C
3. D
4. B
5. B
6. D
7. D
8. D
9. C
10. D

Explanations

1. Given events A and B, use the following formula, defined as the addition rule, to find the probability that A or B will occur.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Define the events.

B = the shirt is blue

S = the shirt has stripes

The question asks for the probability that either event occurs; therefore, the addition rule must be applied.

Rewrite the addition rule for the given events, B and S.

$$P(B \cup S) = P(B) + P(S) - P(B \cap S)$$

Calculate $P(B)$, $P(S)$, and $P(B \cap S)$.

Find $P(B)$. There are 15 total shirts in the closet, and 8 of the shirts are blue.

$$P(B) = \frac{8}{15} \approx 0.53$$

Find $P(S)$. There are 15 total shirts in the closet, and 9 of the total shirts have stripes.

$$P(S) = \frac{9}{15} = 0.6$$

Find $P(B \cap S)$. There are 15 total shirts in the closet, and there are 5 shirts that are blue and have stripes.

$$P(B \cap S) = \frac{5}{15} \approx 0.33$$

Substitute $P(B)$, $P(S)$, and $P(B \cap S)$ into the addition rule formula and simplify to obtain the final answer.

$$\begin{aligned}P(B \cup S) &= P(B) + P(S) - P(B \cap S) \\&= 0.53 + 0.6 - 0.33 \\&= 0.8\end{aligned}$$

2. If the probability of an event occurring is P , then the probability of that event not occurring is $1 - P$.

Since the probability of choosing a comedy movie is $\frac{1}{2}$, the probability of not choosing a comedy movie is given by the following.

$$1 - \frac{1}{2} = \frac{1}{2}$$

3. For each question, there is 1 correct answer choice and 3 incorrect answer choices.

The probability of Bobby randomly guessing on the first two questions and getting the first question incorrect and the second question incorrect is calculated below.

$$P(\text{incorrect, incorrect}) = \frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$$

4. Given that A and B are compound events, use the following conditional probability formula to find the probability that event B will occur given that the event A has already occurred.

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

Define the events.

T = the customer purchased a talk plan

X = the customer purchased a text plan

The question asks for the probability that a customer purchased a text plan given that the customer purchased a talk plan; therefore, the formula for conditional probability must be applied.

Rewrite the formula for conditional probability for the given events, T and X .

$$P(X|T) = \frac{P(T \cap X)}{P(T)}$$

Find $P(T)$. The company has 851 customers, and 768 customers purchased a talk plan.

$$P(T) = \frac{768}{851} \approx 0.9$$

Find $P(T \cap X)$. The company has 851 customers, and 534 of the customers have purchased a talk and text plan.

$$P(T \cap X) = \frac{534}{851} \approx 0.63$$

Substitute $P(T)$ and $P(T \cap X)$ into the formula for conditional probability and simplify to obtain the final answer.

$$\begin{aligned} P(X|T) &= \frac{P(T \cap X)}{P(T)} \\ &= \frac{0.63}{0.9} \\ &\approx 0.7 \end{aligned}$$

5. There are a total of 15 balls in the urn. Of those 15 balls, there are 3 yellow and 2 green, for a total of 5 balls that are either yellow or green.

Therefore, the probability that the ball drawn is either yellow or green is 5 out of 15, or $\frac{1}{3}$.

6. Given that A and B are compound events, use the following formula, defined as the multiplication rule, to find the probability that A *and* B will occur. Remember that $P(B|A)$ is the probability that B will occur given that A has already occurred.

$$P(A \text{ and } B) = P(A)P(B|A)$$

Note: Given that A and B are independent events, the multiplication rule will reduce to the following formula.

$$P(A \text{ and } B) = P(A)P(B)$$

Define the events.

C = the song is country

R = the song is rock

The question asks for the probability that both events occur; therefore, the multiplication rule must be applied.

Rewrite the multiplication rule for the given events, C and R.

$$P(C \text{ and } R) = P(C)P(R|C)$$

Calculate $P(C)$ and $P(R|C)$.

Find $P(C)$. There are 34 songs, and 12 of the total songs are country.

$$P(C) = \frac{12}{34} \approx 0.35$$

Find $P(R|C)$. One song has already been selected; therefore, there are 33 songs remaining, and 10 of the total songs are rock.

$$P(R|C) = \frac{10}{33} \approx 0.3$$

Substitute $P(C)$ and $P(R|C)$ into the multiplication rule formula and simplify to obtain the final answer.

$$\begin{aligned} P(C \text{ and } R) &= P(C)P(R|C) \\ &= 0.35 \times 0.3 \\ &= 0.11 \end{aligned}$$

7. There are 25 students on the Honor Roll. If 16 students are boys, then 9 students are girls.

So, the probability of choosing a girl's name is $\frac{9}{25}$.

8. To determine the probability that Jack will have to do at least one of the chores, find the complement event, and then subtract its probability from one.

In this case, the complement event would be for all of Jack's siblings to do the two chores. The probability of that happening is shown below.

$$\begin{aligned} P(\text{complement}) &= \left(\frac{3}{4}\right)^2 \\ &= \frac{9}{16} \end{aligned}$$


Now, subtract this probability from one.

$$\begin{aligned} 1 - P(\text{complement}) &= 1 - \frac{9}{16} \\ &= \frac{16}{16} - \frac{9}{16} \\ &= \frac{7}{16} \end{aligned}$$

Therefore, the probability that Jack will have to do at least one of the chores is $\frac{7}{16}$.

9. Remember there are five times as many whitetail deer as mule deer.

So, for example, if there were 5 mule deer, then there would be 25 whitetail deer.

Out of the 30 deer in the woods, there would be 5 mule deer. So, the probability of getting a mule deer would be , which reduces to $\frac{1}{6}$.

10. Since the order in which the cards are drawn does not matter, the combination equation will be used.

$$\binom{n}{k} = \frac{n!}{(n-k)!k!}$$

First, determine the number of ways that 5 cards can be drawn.

$$\begin{aligned}\binom{52}{5} &= \frac{52!}{(52-5)!5!} \\ &= \frac{52!}{47!5!} \\ &= \frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48 \cdot 47!}{47!5!} \\ &= \frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} \\ &= \frac{311,875,200}{120} \\ &= 2,598,960\end{aligned}$$

Next, determine the number of ways that 3 cards can have the same rank. This would be the number of ranks multiplied by the number of ways that 3 cards can be chosen from each rank. There are 13 ranks with 4 cards in each rank.

$$13 \cdot \binom{4}{3} = 13 \cdot 4 = 52$$

Then, calculate the number of ways 2 cards can be drawn from the remaining 12 ranks. These cards must have equivalent rank but it must be a different rank than that of the first 3 cards.

$$12 \cdot \binom{4}{2} = 12 \cdot 6 = 72$$

Multiply these two values together to determine the number of ways that this situation can occur.

$$52 \cdot 72 = 3,744$$

Finally, calculate the probability.

$$\begin{aligned}P(3 \text{ cards of one rank and 2 of another}) &= \frac{3,744}{2,598,960} \\ &= \frac{6}{4,165}\end{aligned}$$